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Ray W Ogden: An Appreciation

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This special issue of *Mathematics and Mechanics of Solids* is dedicated to Professor Ray Ogden FRS on the occasion of his 70th birthday. It is a companion volume to another special issue edited by our colleagues Roger Bustamente, Jose Merodio and David Steigmann at the *IMA Journal of Applied Mathematics*.

Ray Ogden's work has had a major influence in the broad field of solid mechanics, within the context of continuum mechanics. It continues to do so as can be checked by looking at the exponential rise of his citation count, totaling according to Google Scholar more than 15,000 to date, with an *h*-index of 51. Whatever value we attach to bibliometric indicators, these numbers clearly point to a deep and profound impact. Here, instead of presenting the long list of his achievements, awards and publications (to be found elsewhere), we prefer to highlight three of the themes for which his work has received the most attention. Needless to say, the spectrum of his abilities is far wider.

His first major breakthrough has been the elaboration of what is now universally called the *Ogden model for rubber-like materials*. It is the outcome of a methodical and rational examination of the Valandis–Landel hypothesis for rubber experiments. The resulting strain energy density W reads [1, 2]

$$W = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right),$$

where the λ are the principal stretches of deformation and α_i, μ_i are phenomenological material parameters, to be determined from a nonlinear least-squares curve fitting exercise. Here N is an integer number which can be increased if one wishes to meet some assigned precision criterion in the curve fitting exercise. This versatility makes the Ogden material very popular with experimentalists and engineers, as attested by its implementation in all major finite element (FE) codes. Some codes even perform the curve fitting exercise for its users, who are simply required to provide the experimental data from a tensile test performed on a given soft material. As it happens, non-unicity of the best-fit parameter set can occur when $N \geq 2$. This potential problem was identified and addressed by Ray Ogden himself (in collaboration with Giuseppe Saccomandi and Ivonne Sgura [3]) thus demonstrating, if needs be, that he is true to the very spirit of science, which is to always question and improve on past results.

A second easily identifiable landmark paper is his 2000 collaboration with Gerard Holzapfel and Christian Gasser on the *mechanical modeling of arteries* [4]. Having proven that it should at least incorporate hyperelasticity, residual stresses, inhomogeneity and orthotropy, they proposed what is now considered the gold standard of strain energies for fiber-reinforced biological soft tissues,

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$$W = \frac{\mu}{2}(I_1 - 3) + \frac{k_1}{2k_2}\left[\mathrm{e}^{k_2(I_4-1)^2} + \mathrm{e}^{k_2(I_6-1)^2} - 2\right].$$

Here the first term describes the isotropic neo-Hookean matrix into which the collagen fibers are imbedded. The fibers' higher stiffness and strain-hardening properties are captured by the next two terms, which give an exponential increase of the energy and of the stress with the squared stretches in the directions of the two families of fibers I_4, I_6. This model is also implemented in all major FE codes. In a series of most insightful follow-up papers [5–9], always guided by biology and experiments, the authors incorporate fiber dispersion and strain-hardening of the isotropic matrix. To date, the original 2000 article has received more than 1600 citations.

The third theme we like to highlight is Ray Ogden's contribution, over the last 10 years, to modeling the nonlinear theories describing *electro- and magneto-mechanical couplings in elastomeric and polymeric materials*. The developed theories of magnetoelasticity [10] and electroelasticity [11] generalize the theories of electromagnetism and nonlinear solid mechanics and provide the foundation of much of the recent developments in this area. Using the connections between Eulerian and Lagrangian forms of magnetic and electric field quantities and introducing an amended free energy function, the structure of the constitutive equations assumes a very elegant form. For example, when a soft, unconstrained magneto-sensitive material is deformed and subjected to a magnetic induction, the total nominal stress tensor $\mathbf{T}$ and the Lagrangian form of the magnetic field vector $\mathbf{H}_\ell$ are given by the simple relations [10]

$$\mathbf{T} = \frac{\partial \Omega}{\partial \mathbf{F}}, \quad \mathbf{H}_\ell = \frac{\partial \Omega}{\partial \mathbf{B}_\ell},$$

where the potential function Ω depends on the deformation gradient $\mathbf{F}$ and on the Lagrangian form of the magnetic induction vector $\mathbf{B}_\ell$. The development in [11] provides similar elegant relations to describe the response of soft dielectric materials to the application of an electric field. The variational statements of these theories are given in [12, 13], the incremental equations and aspects of instability are presented in [14–17]. Significant new results are contained in the papers [18–21], which address the propagation of waves in nonlinear magnetoelastic and electroelastic materials. The fundamental concepts of the nonlinear theory of electroelastic and magnetoelastic interactions, including solutions of boundary value problems, is presented in the recently published monograph [22].

The collection of papers collected in this special issue touches upon many aspects of Ray's work. The paper by Valentina Balbi and Pasquale Ciarletta deals with the problem of the torsional instability of a hollow tube under internal pressure, a problem which had been left untouched until now, and with obvious repercussions for the mechanics of veins and the modeling of anastomosis. The article by Salvatore Federico, Alfio Grillo and Shoji Imatani is interested in the implementation of strict and slight incompressibility in anisotropic elasticity, a problem that Ray Ogden recently approached too [23, 24]. The contribution by Min Wu and Martine Ben Amar looks at

Figure 1. Ray Ogden in Santiago de Chile, January 2011.

how growing disks of biological soft tissues should be modeled, with particular attention paid to the role played by the anisotropic growth and anisotropic invariants. The study by Christian Cyron and Jay Humphrey looks at preferred fiber orientations in healthy arteries and veins: there the authors extend the usual hypothesis of stress and strain optimization to the concept of vessel stability and optimization of wall thickness/volume and pulse pressure of the vasculature. The paper by Igor Novak and Lev Truskinovsky is a theoretical investigation of some unexplained observations of skeletal muscle dynamics on the descending limb of the force–length relationship. The article by Noy Chen and Gal deBotton is concerned with the modeling of electroactive polymers, and implements the hypotheses of long-chain molecules and Gaussian distribution to derive a macroscopic response function. Morteza Hakimi Siboni, Reza Avazmohammadi and Pedro Ponte Castañeda investigate loss of positive definiteness and loss of strong ellipticity of dielectric elastomer composites subjected to dead electromechanical loading. The composites consist of a dielectric elastomer matrix and aligned, rigid-dielectric fibers of elliptical cross-section randomly distributed in the transverse plane. The paper by Quoc Son Nguyen provides new insights into some aspects of gradient damage and plasticity models in solid mechanics. In their article, Lior Falach and Reuven Segev propose a formulation of the Reynolds transport theorem in terms of currents, by considering field theories where forms represent some extensive properties over a manifold. Finally, a further paper by Patrizio Neff, Ingo Münch and Robert Martin investigates the origins of the use of the logarithmic strain for a nonlinear stress–strain relation and its performance (compared with the Ogden model) in the multi-axial testing of rubber. That latter article was too long for inclusion here and it is published in another issue of the journal. Of course none of these papers could have been included in this special issue were it not for the diligence and professionalism of the reviewers, and we are all deeply grateful to them for providing such a first-class service.

In an increasingly crowded and competitive academic environment, one can sometimes get overwhelmed with the feeling that "the race is not to the swift [...] nor yet favour to men of skill", but that some scientists rise to prominence thanks to "time and chance". That is why we feel privileged to have been able to witness that the phenomenal accomplishments of Professor Ray Ogden were arrived at by sheer scientific curiosity, encyclopedic mathematical knowledge, deep insights into continuum mechanics and an amazing output of the best quality books, chapters, articles, courses, conferences and talks. Most important of all, all of those who have ever met Ray can testify to his gentleness, patience and helpfulness. His path is truly an inspirational blueprint to all of us. In conclusion, we can take comfort in the notion that nice guys can also finish first, as Ray has superbly demonstrated with his career so far. May it go on for long!

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