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Surface acoustic waves in rotating orthorhombic crystals

By Michel Destrade

Laboratoire de Modélisation en Mécanique, UMR 7607, CNRS, Université Pierre et Marie Curie, 4 place Jussieu, Tour 66, case 162, 75252 Paris Cedex 05, France

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The propagation of surface (Rayleigh) waves over a rotating orthorhombic crystal is studied. The crystal possesses three crystallographic axes, normal to the symmetry planes: the half-space is cut along a plane normal to one of these axes, the wave travels in the direction of another, and the rotation occurs at a uniform rate about any of the three axes. The secular equation for the surface wave speed is found explicitly; in contrast to the non-rotating case, it is dispersive (frequency dependent). Both Coriolis and centrifugal accelerations appear in the equations of motion: none can be neglected in favour of the other, even at small rotation rates.

Keywords: surface waves; rotating media; orthotropic crystals; centrifugal and Coriolis accelerations; dispersion equation

1. Introduction

The main applications of surface acoustic wave (SAW) devices are in the field of wireless communication, where SAW components are used to generate and collect high-frequency signals. In mobile phones, global positioning systems or colour television sets, these components are of course motionless. There exist, however, situations where SAWs propagate over a rotating surface. For instance, SAW sensors have been used to monitor the pressure of a tyre in a moving automobile (Pohl *et al*. 1997); an ultrasonic SAW-based apparatus was designed to detect flaws on the surface of a rotating member such as a roller bearing (Kawasaki *et al*. 1991); and robust and shock-resistant SAW resonator gyroscopes have been miniaturized to a 1 cm $\times$ 1 cm surface (Jose *et al*. 2002). For the theoretical aspect, Schoenberg & Censor (1973) seem to have been the first authors to describe Rayleigh waves in a rotating, homogeneous, isotropic, linear, elastic medium for any orientation of the rotation axis with respect to the free surface, and with both the Coriolis and the centrifugal accelerations taken into account. They pointed out that, in contrast to the non-rotating case, bulk and surface waves are dispersive, and that the acoustical tensor is Hermitian instead of real symmetric; they did not, however, derive the secular equation for surface waves. Clarke & Burdess (1994) obtained this equation for an isotropic half-space rotating about an axis orthogonal to the direction of propagation and to the normal to the half-space. Others (see, for example, Lao 1980; Wauer 1999; Grigor'evskiĭ *et al*. 2000) considered similar problems but neglected the centrifugal acceleration. For anisotropic crystals, Fang *et al*. (2000) and Zhou & Jiang (2001) considered crystals of tetragonal symmetry but did not derive the secular equation explicitly. As pointed

out in the latter article, the influence of the centrifugal acceleration upon the SAW speed is of the same order as that of the Coriolis acceleration and should not be overlooked, even at small rotation rates. In this paper, an analytical investigation of the propagation of SAWs in a rotating orthorhombic crystal is presented.

In § 2, the crystal considered is at first of general (triclinic) anisotropy and rotates at a uniform rate about any axis. The equations of motion and the boundary conditions are derived. Then the analysis is specialized to crystals with rhombic and higher symmetries. The material is cut along the plane of symmetry $x_2 = 0$ and the wave travels in the $\mathrm{O}x_1$-direction, where the $\mathrm{O}x_i$-axes are crystallographic axes. The semi-infinite body rotates along $\mathrm{O}x_1$, $\mathrm{O}x_2$ or $\mathrm{O}x_3$. In the latter case, anti-plane motions and stresses decouple from their in-plane counterparts; in the other two cases, they do not. The choice of orthorhombic symmetry is of great importance because, as pointed out by Royer & Dieulesaint (1984), it covers 16 types of symmetries, including tetragonal, hexagonal, cubic and of course isotropic; the choice of a rotation axis aligned with the normal to the free plane or to crystallographic axes in this plane is justified from an experimental point of view. In § 3, the secular (dispersion) equations are derived explicitly for each rotation, using the method of the polarization vector. This method was used by Currie (1979) and Taziev (1989) in order to derive explicit secular equations for SAWs in non-rotating crystals.

Recently, Ting (2004*a*, *b*) improved upon this method and placed it within the framework of the Stroh (1958, 1962) formalism. Here, the fundamental equations of the method are obtained in a manner which does not rely upon this formalism. The secular equation is obtained as a quadratic relationship between three determinants (of 2×2 matrices for rotation about $\mathrm{O}x_3$, of 3×3 matrices for rotations about $\mathrm{O}x_1$ or $\mathrm{O}x_2$), whose elements are given explicitly in terms of the elastic parameters, the wave speed and δ—the ratio of the rotation rate by the wave frequency. Several secular equations arise in the resolution of the problem but only one is kept; the others are dismissed on the argument that they do not reduce to the known single cubic secular equation when the rotation rate vanishes. Numerically, the Rayleigh wave speed decreases monotonically as δ increases. This influence is shown graphically for α-iodic acid (HIO_3, rhombic). For this graph, δ is chosen to vary from 0 to 10. Although it is not realistic to imagine a crystal rotating at a frequency which would be greater, or even close to, the frequency of an ultrasonic surface wave, this wide range is chosen to show numerically that the wave speed tends to zero with increasing δ without, seemingly, ever reaching it. For silica (SiO_2, isotropic), the calculations are conducted for a smaller range ($0.0 \leqslant \delta \leqslant 0.1$) with and without the contribution of the centrifugal acceleration in order to show its importance.

Some theoretical issues, such as the questions of existence and uniqueness of a surface wave in a rotating crystal or the question of the general behaviour of the wave speed as a function of δ, are left open.

2. Equations of motion

(*a*) *General anisotropy and arbitrary rotation axis*

Consider a semi-infinite body made of a linearly elastic, anisotropic crystal. For two-dimensional deformations, the stress–strain relations are (Ting 1996, p. 38)

$$\boldsymbol{\sigma}^{\mathrm{o}} = \boldsymbol{C}^{\mathrm{o}}\boldsymbol{\varepsilon}^{\mathrm{o}} \quad \text{or} \quad \boldsymbol{\varepsilon}^{\mathrm{o}} = \boldsymbol{s}'\boldsymbol{\sigma}^{\mathrm{o}}, \quad \boldsymbol{C}^{\mathrm{o}}\boldsymbol{s}' = \boldsymbol{I}, \tag{2.1}$$

where

$$\boldsymbol{\sigma}^{\mathrm{o}} = [\sigma_{11}, \sigma_{22}, \sigma_{23}, \sigma_{31}, \sigma_{12}]^{\mathrm{T}}, \qquad \boldsymbol{\varepsilon}^{\mathrm{o}} = [\epsilon_{11}, \epsilon_{22}, 2\epsilon_{23}, 2\epsilon_{31}, 2\epsilon_{12}]^{\mathrm{T}}, \tag{2.2}$$

and

$$\boldsymbol{C}^{\mathrm{o}} = \begin{bmatrix} C_{11} & C_{12} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{24} & C_{25} & C_{26} \\ & & C_{44} & C_{45} & C_{46} \\ & & & C_{55} & C_{56} \\ & & & & C_{66} \end{bmatrix}, \qquad \boldsymbol{s}' = \begin{bmatrix} s'_{11} & s'_{12} & s'_{14} & s'_{15} & s'_{16} \\ & s'_{22} & s'_{24} & s'_{25} & s'_{26} \\ & & s'_{44} & s'_{45} & s'_{46} \\ & & & s'_{55} & s'_{56} \\ & & & & s'_{66} \end{bmatrix}. \tag{2.3}$$

Here the C_{ij} are derived, using the Voigt (1910, p. 560) contracted notation, from the fourth-order elastic stiffness tensor and the reduced elastic compliances s'_{ij} are defined by $(2.1)_3$.

In the Cartesian orthogonal coordinate system $(\mathrm{O}x_1x_2x_3) \equiv (\mathrm{O}\boldsymbol{ijk})$, where $x_2 \geqslant 0$ is the region occupied by the half-space, a surface (Rayleigh) wave travelling at speed v and wavenumber k in the x_1-direction with attenuation in the x_2-direction is described by the following mechanical displacement field,

$$\boldsymbol{u}(x_1, x_2, x_3, t) = \boldsymbol{U}(kx_2)\mathrm{e}^{\mathrm{i}k(x_1 - vt)}, \quad \boldsymbol{U}(\infty) = \boldsymbol{0}. \tag{2.4}$$

From $(2.1)_1$ it follows that the corresponding stress components are of similar form. In particular, the tractions acting upon the planes $x_2 = \text{const.}$ can be written as

$$\sigma_{i2}(x_1, x_2, x_3, t) = \mathrm{i}kt_i(kx_2)\mathrm{e}^{\mathrm{i}k(x_1 - vt)}. \tag{2.5}$$

Now consider that the crystal is rotating at a uniform rate Ω about the direction of a unit vector $\boldsymbol{w}$. Then the incremental (time-dependent) equations of motion in the rotating frame are (Schoenberg & Censor 1973)

$$\operatorname{div} \boldsymbol{\sigma} = \rho\ddot{\boldsymbol{u}} + 2\rho\Omega\boldsymbol{w} \times \dot{\boldsymbol{u}} + \rho\Omega^2\boldsymbol{w} \times (\boldsymbol{w} \times \boldsymbol{u}), \tag{2.6}$$

where ρ is the mass density and the dot denotes differentiation with respect to time. Note that on the right-hand side of the equations both the Coriolis (second term) and the centripetal (third term) accelerations appear. Using (2.1), (2.4) and (2.5), these equations can be formulated as a linear homogeneous system of first-order differential equations for $\boldsymbol{U} = [U_1, U_2, U_3]^{\mathrm{T}}$ and $\boldsymbol{t} = [t_1, t_2, t_3]^{\mathrm{T}}$:

$$\begin{bmatrix} \boldsymbol{U}' \\ \boldsymbol{t}' \end{bmatrix} = \mathrm{i} \begin{bmatrix} \boldsymbol{N}_1 & \boldsymbol{N}_2 \\ \boldsymbol{N}_3 + \check{\boldsymbol{J}} + X\boldsymbol{1} & \boldsymbol{N}_1^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{t} \end{bmatrix}, \tag{2.7}$$

where $X = \rho v^2$ and the prime denotes differentiation with respect to kx_2. Here, $\boldsymbol{N}_1$, $\boldsymbol{N}_2 = \boldsymbol{N}_2^{\mathrm{T}}$ and $\boldsymbol{N}_3 = \boldsymbol{N}_3^{\mathrm{T}}$ are the usual submatrices of the fundamental matrix $\boldsymbol{N}$ (Ingebrigsten & Tonning 1969). Barnett & Chadwick (1990) and Ting (1988) give explicit expressions of the components of the $\boldsymbol{N}_i$ in terms of the C_{ij} and of the s'_{ij}, respectively. The matrix $\check{\boldsymbol{J}}$ contains all the information relative to the rotation. Explicitly (Schoenberg & Censor 1973),

$$\check{J}_{ij} = [(\delta_{ij} - w_iw_j)\delta^2 - 2\mathrm{i}\epsilon_{ijk}w_k\delta]X, \quad \delta = \Omega/\omega, \tag{2.8}$$

where $\omega = kv$ is the real frequency of the wave, δ_{ij} is the Kronecker operator and ϵ_{ijk} is the alternator. Note that $\check{\boldsymbol{J}}$ is Hermitian.

Finally, the following boundary conditions apply: vanishing of the tractions on the free plane surface $x_2 = 0$ and of the wave as $x_2 \to \infty$, that is,

$$\boldsymbol{t}(0) = \boldsymbol{0}, \qquad \boldsymbol{U}(\infty) = \boldsymbol{0}, \qquad \boldsymbol{t}(\infty) = \boldsymbol{0}. \tag{2.9}$$

(*b*) *Orthorhombic crystals*

Henceforward, we are concerned with orthorhombic crystals whose crystallographic axes are aligned with the $\mathrm{O}x_i$. In that case, we have

$$\boldsymbol{N}_1 = \begin{bmatrix} 0 & -1 & 0 \\ -\dfrac{s'_{12}}{s'_{11}} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \boldsymbol{N}_2 = \begin{bmatrix} s'_{66} & 0 & 0 \\ 0 & s'_{22} - \dfrac{s'^2_{12}}{s'_{11}} & 0 \\ 0 & 0 & s'_{44} \end{bmatrix}. \tag{2.10}$$

Also, the crystal is assumed to be rotating about a crystallographic axis. Introducing the Hermitian tensor $\check{\boldsymbol{K}}^{(1)} = \boldsymbol{N}_3 + \check{\boldsymbol{J}} + X\mathbf{1}$ (lower left submatrix in (2.7)), we have

$$\check{\boldsymbol{K}}^{(1)} = \begin{bmatrix} X - \dfrac{1}{s'_{11}} & 0 & 0 \\ 0 & (1+\delta^2)X & -2\mathrm{i}\delta X \\ 0 & 2\mathrm{i}\delta X & (1+\delta^2)X - \dfrac{1}{s'_{55}} \end{bmatrix},$$

$$\begin{bmatrix} (1+\delta^2)X - \dfrac{1}{s'_{11}} & 0 & 2\mathrm{i}\delta X \\ 0 & X & 0 \\ -2\mathrm{i}\delta X & 0 & (1+\delta^2)X - \dfrac{1}{s'_{55}} \end{bmatrix},$$

$$\begin{bmatrix} (1+\delta^2)X - \dfrac{1}{s'_{11}} & 2\mathrm{i}\delta X & 0 \\ -2\mathrm{i}\delta X & (1+\delta^2)X & 0 \\ 0 & 0 & X - \dfrac{1}{s'_{55}} \end{bmatrix}, \tag{2.11}$$

for rotation about the $\mathrm{O}x_1$-axis, $\mathrm{O}x_2$-axis and $\mathrm{O}x_3$-axis, respectively.

3. Explicit dispersion equations

The Stroh–Barnett–Lothe formalism has beautifully addressed and resolved most of the problems arising in linear anisotropic elasticity (see the textbook by Ting (1996) for a thorough review). Some researchers have, however, followed other routes for the study of SAWs in anisotropic crystals (see Ting (2004*b*) for an exposition of several methods). For instance, Fu & Mielke (2002) recently proposed a highly efficient numerical scheme to determine the Rayleigh wave speed, which is based on a Ricatti equation. Here the method of the polarization vector (Currie 1979; Taziev 1989; Ting 2004*a*, *b*) is presented for rotating orthorhombic crystals. Only simple algebraic manipulations and first integrals (Mozhaev 1995) are used, so the derivation should appeal to the reader who is not familiar with the Stroh (1958, 1962) formalism and with the orthogonality relations (Barnett & Lothe 1973).

(*a*) *Method of the polarization vector*

Let us examine the equations of motion (2.7) closer. They read

$$-\mathrm{i}\boldsymbol{U}' = \boldsymbol{N}_1\boldsymbol{U} + \boldsymbol{N}_2\boldsymbol{t}, \qquad -\mathrm{i}\boldsymbol{t}' = \check{\boldsymbol{K}}^{(1)}\boldsymbol{U} + \boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{t}. \tag{3.1}$$

Take the scalar product of the first line by $\bar{\boldsymbol{t}}'$ and of the second line by $\overline{\boldsymbol{U}}'$; then add, together with the complex conjugate; finally, integrate between 0 and ∞ to obtain

$$[\overline{\boldsymbol{U}} \cdot \check{\boldsymbol{K}}^{(1)}\boldsymbol{U} + \bar{\boldsymbol{t}} \cdot \boldsymbol{N}_2\boldsymbol{t} + \bar{\boldsymbol{t}} \cdot \boldsymbol{N}_1\boldsymbol{U} + \boldsymbol{t} \cdot \boldsymbol{N}_1\overline{\boldsymbol{U}}]_0^\infty = 0, \tag{3.2}$$

which, using the boundary conditions (2.9), reduces to $\overline{\boldsymbol{U}}(0) \cdot \check{\boldsymbol{K}}^{(1)}\boldsymbol{U}(0) = 0$, a relation first established by Stroh (1958) in the static, non-rotating case.

This procedure is easily generalized as follows. First compute $\boldsymbol{N}$ (the 6×6 matrix on the right-hand side of the equations of motion (2.7)) to the power n, where n is any positive or negative integer, as

$$\boldsymbol{N}^n = \begin{bmatrix} \boldsymbol{N}_1^{(n)} & \boldsymbol{N}_2^{(n)} \\ \check{\boldsymbol{K}}^{(n)} & \boldsymbol{N}_1^{(n)\mathrm{T}} \end{bmatrix}, \tag{3.3}$$

(say). Of course, the matrices $\boldsymbol{N}_1^{(n)}$, $\boldsymbol{N}_2^{(n)}$ and $\check{\boldsymbol{K}}^{(n)}$ do not coincide with $\boldsymbol{N}_1^n$, $\boldsymbol{N}_2^n$, and $\check{\boldsymbol{K}}^n$, respectively. The matrix $\boldsymbol{N}_2^{(n)}$ is, however, symmetric just like $\boldsymbol{N}_2$ is, as can be proved by induction (Ting 2004*a*). Similarly, it can be proved, in essentially the same manner, that $\check{\boldsymbol{K}}^{(n)}$ is Hermitian just like $\check{\boldsymbol{K}}^{(1)}$ is. Now multiply (2.7) in turn by $\boldsymbol{N}^{-1}$ and by $-\mathrm{i}\boldsymbol{N}$ to get

$$\left.\begin{aligned} \mathrm{i}\boldsymbol{U} &= \boldsymbol{N}_1^{(-1)}\boldsymbol{U}' + \boldsymbol{N}_2^{(-1)}\boldsymbol{t}', & -\mathrm{i}\boldsymbol{N}_1\boldsymbol{U}' - \mathrm{i}\boldsymbol{N}_2\boldsymbol{t}' &= \boldsymbol{N}_1^{(2)}\boldsymbol{U} + \boldsymbol{N}_2^{(2)}\boldsymbol{t}, \\ \mathrm{i}\boldsymbol{t} &= \check{\boldsymbol{K}}^{(-1)}\boldsymbol{U}' + \boldsymbol{N}_1^{(-1)\mathrm{T}}\boldsymbol{t}', & -\mathrm{i}\check{\boldsymbol{K}}^{(1)}\boldsymbol{U}' - \mathrm{i}\boldsymbol{N}_1^{\mathrm{T}}\boldsymbol{t}' &= \check{\boldsymbol{K}}^{(2)}\boldsymbol{U} + \boldsymbol{N}_1^{(2)\mathrm{T}}\boldsymbol{t}. \end{aligned}\right\} \tag{3.4}$$

Take the scalar product of the first line in the first (second) system by $\bar{\boldsymbol{t}}$ ($\bar{\boldsymbol{t}}'$) and of the second line by $\overline{\boldsymbol{U}}$ ($\overline{\boldsymbol{U}}'$); then add, together with the complex conjugate; finally, integrate between 0 and ∞ to obtain

$$\left.\begin{aligned} [\overline{\boldsymbol{U}} \cdot \check{\boldsymbol{K}}^{(-1)}\boldsymbol{U} + \bar{\boldsymbol{t}} \cdot \boldsymbol{N}_2^{(-1)}\boldsymbol{t} + \bar{\boldsymbol{t}} \cdot \boldsymbol{N}_1^{(-1)}\boldsymbol{U} + \boldsymbol{t} \cdot \boldsymbol{N}_1^{(-1)}\overline{\boldsymbol{U}}]_0^\infty = 0, \\ [\overline{\boldsymbol{U}} \cdot \check{\boldsymbol{K}}^{(2)}\boldsymbol{U} + \bar{\boldsymbol{t}} \cdot \boldsymbol{N}_2^{(2)}\boldsymbol{t} + \bar{\boldsymbol{t}} \cdot \boldsymbol{N}_1^{(2)}\boldsymbol{U} + \boldsymbol{t} \cdot \boldsymbol{N}_1^{(2)}\overline{\boldsymbol{U}}]_0^\infty = 0, \end{aligned}\right\} \tag{3.5}$$

respectively. Using the boundary conditions (2.9), we have $\overline{\boldsymbol{U}}(0) \cdot \check{\boldsymbol{K}}^{(-1)}\boldsymbol{U}(0) = 0$ and $\overline{\boldsymbol{U}}(0) \cdot \check{\boldsymbol{K}}^{(2)}\boldsymbol{U}(0) = 0$. These steps may be repeated *ad infinitum* for any positive or negative power n of $\boldsymbol{N}$ to give

$$\overline{\boldsymbol{U}}(0) \cdot \check{\boldsymbol{K}}^{(n)}\boldsymbol{U}(0) = 0. \tag{3.6}$$

However, because of the Cayley–Hamilton theorem, only five $\boldsymbol{N}^n$ are linearly independent and, consequently, (3.6) yields at most five equations. In a non-rotating frame, Currie (1979) used the equations written at $n = 1, 2, 3$ for SAWs in the plane of symmetry of a monoclinic crystal and Taziev (1989) used the equations written at $n = 1, 2, 3, 4, 5$ for triclinic crystals. Recently, Ting (2004*a*) pointed out that the choices $n = -2, -1, 1, 2, 3$ lead to simpler expressions.

(*b*) *Rotation about* $\mathrm{O}x_1$ *(propagation direction)*

When the orthorhombic crystal rotates about the propagation direction $\mathrm{O}x_1$, the matrices $\check{\boldsymbol{K}}^{(-2)}$ and $\check{\boldsymbol{K}}^{(2)}$ have the following structure:

$$\check{\boldsymbol{K}}^{(n)} = \begin{bmatrix} 0 & \check{K}_{12}^{(n)} & \mathrm{i}K_{13}^{(n)} \\ \check{K}_{12}^{(n)} & 0 & 0 \\ -\mathrm{i}K_{13}^{(n)} & 0 & 0 \end{bmatrix}, \quad n = -2, 2, \tag{3.7}$$

where $\check{K}_{12}^{(n)}$ and $K_{13}^{(n)}$ are *real*. Explicitly,

$$s'_{11}\check{K}_{12}^{(2)} = 1 - [s'_{11} + s'_{12}(1+\delta^2)]X, \qquad s'_{11}K_{13}^{(2)} = 2s'_{12}\delta X, \tag{3.8}$$

and

$$\left.\begin{aligned} D_1\check{K}_{12}^{(-2)} &= -1 + [s'_{11} - (s'_{12}+s'_{55})(1+\delta^2)]X \\ &\qquad + [s'_{11}(1+\delta^2) + s'_{12}(1-\delta^2)^2]s'_{55}X^2, \\ s'_{44}D_1K_{13}^{(-2)} &= 2s'_{55}\delta X, \\ D_1 &= 1 - (s'_{55}+s'_{66})(1+\delta^2)X + (1-\delta^2)^2 s'_{55}s'_{66}X^2. \end{aligned}\right\} \tag{3.9}$$

Equations (3.6) written at $n=-2,2$, for $\boldsymbol{U}(0) = [1, \alpha_1 + \mathrm{i}\alpha_2, \beta_1 + \mathrm{i}\beta_2]^{\mathrm{T}}$, give

$$\check{K}_{12}^{(-2)}\alpha_1 - K_{13}^{(-2)}\beta_2 = 0, \qquad \check{K}_{12}^{(2)}\alpha_1 - K_{13}^{(2)}\beta_2 = 0. \tag{3.10}$$

Hence, either (a) $\check{K}_{12}^{(-2)}K_{13}^{(2)} - \check{K}_{12}^{(2)}K_{13}^{(-2)} = 0$ or (b) $\alpha_1 = \beta_2 = 0$. The first possibility is not a valid secular equation, because it is not consistent with the non-rotating case; in other words, (a) at $\delta = 0$ does not reduce to the cubic Rayleigh function for non-rotating orthorhombic crystals:

$$1 - (2s'_{11}+s'_{66})X + s'_{11}(s'_{11} - s'_{22} + 2s'_{66})X^2 + s'_{11}(s'_{11}s'_{22} - s'^2_{12} - s'_{11}s'_{66})X^3 = 0. \tag{3.11}$$

Thus (b) applies and

$$\boldsymbol{U}(0) = [1, \mathrm{i}\alpha_2, \beta_1]^{\mathrm{T}}. \tag{3.12}$$

The directions of the vectors $\boldsymbol{i} + \beta_1\boldsymbol{k}$ and $\alpha_2\boldsymbol{j}$ are conjugate directions with respect to the polarization ellipse of the wave at the free surface (Boulanger & Hayes 1993). In fact, because they are orthogonal, they are along the principal axes of the ellipse. Now we compute α_2 and β_1.

The matrices $\check{\boldsymbol{K}}^{(-1)}$, $\check{\boldsymbol{K}}^{(1)}$ and $\check{\boldsymbol{K}}^{(3)}$ have the following structure:

$$\check{\boldsymbol{K}}^{(n)} = \begin{bmatrix} \check{K}_{11}^{(n)} & 0 & 0 \\ 0 & \check{K}_{22}^{(n)} & \mathrm{i}K_{23}^{(n)} \\ 0 & -\mathrm{i}K_{23}^{(n)} & \check{K}_{33}^{(n)} \end{bmatrix}, \quad n = -1, 1, 3, \tag{3.13}$$

where $\check{K}_{11}^{(n)}$, $\check{K}_{22}^{(n)}$, $\check{K}_{33}^{(n)}$ and $K_{23}^{(n)}$ are *real*. Explicitly,

$$\left.\begin{aligned} D_1\check{K}_{11}^{(-1)} &= -[1 + \delta^2 - s'_{55}(1-\delta^2)^2X]X, & s'_{44}\check{K}_{33}^{(-1)} &= 1, \\ [s'_{22} - (s'_{11}s'_{22} - s'^2_{12})X]\check{K}_{22}^{(-1)} &= 1 - s'_{11}X, & K_{23}^{(-1)} &= 0, \end{aligned}\right\} \tag{3.14}$$

where D_1 is defined in $(3.9)_3$; the components of $\check{\boldsymbol{K}}^{(1)}$ are given in $(2.11)_1$; and

$$\left.\begin{aligned} s'_{11}\check{K}_{11}^{(3)} &= \frac{s'_{66}}{s'_{11}} - 2\frac{s'_{12}}{s'_{11}} + \left[2(s'_{12} - s'_{66}) + \frac{s'^2_{12}}{s'_{11}}(1+\delta^2)\right]X + s'_{11}s'_{66}X^2, \\ s'_{11}\check{K}_{22}^{(3)} &= -1 + [s'_{11} + 2s'_{12}(1+\delta^2)]X + [(s'_{11}s'_{22} - s'^2_{12})(1+\delta^2)^2 + 4s'_{11}s'_{44}\delta^2]X^2, \\ s'_{55}\check{K}_{33}^{(3)} &= \frac{s'_{44}}{s'_{55}} - 2s'_{44}(1+\delta^2)X + s'_{55}\left[s'_{44}(1+\delta^2)^2 + 4\left(s'_{22} - \frac{s'^2_{12}}{s'_{11}}\right)\delta^2\right]X^2, \\ s'_{11}K_{23}^{(3)} &= -2\delta X\left[s'_{12} - s'_{11}\frac{s'_{44}}{s'_{55}} + (s'_{11}s'_{22} - s'^2_{12} + s'_{11}s'_{44})(1+\delta^2)X\right]. \end{aligned}\right\} \tag{3.15}$$

Substitution of (3.12) into equations (3.6) written at $n=-1,1,3$ gives

$$\check{K}_{11}^{(n)}+\check{K}_{33}^{(n)}\beta_1^2+\check{K}_{22}^{(n)}\alpha_2^2+2K_{23}^{(n)}\alpha_2\beta_1=0,\quad n=-1,1,3. \tag{3.16}$$

These three equations are rewritten as $F_{ik}g_k=-h_i$ with

$$\boldsymbol{F}=\begin{bmatrix}\check{K}_{33}^{(-1)} & \check{K}_{22}^{(-1)} & K_{23}^{(-1)}\\ \check{K}_{33}^{(1)} & \check{K}_{22}^{(1)} & K_{23}^{(1)}\\ \check{K}_{33}^{(3)} & \check{K}_{22}^{(3)} & K_{23}^{(3)}\end{bmatrix},\qquad \boldsymbol{g}=\begin{bmatrix}\beta_1^2\\ \alpha_2^2\\ 2\alpha_2\beta_1\end{bmatrix},\qquad \boldsymbol{h}=\begin{bmatrix}\check{K}_{11}^{(-1)}\\ \check{K}_{11}^{(1)}\\ \check{K}_{11}^{(3)}\end{bmatrix}. \tag{3.17}$$

Let $\Delta=\det\boldsymbol{F}$, and Δ_k be the determinant of the matrix obtained from $\boldsymbol{F}$ by replacing the kth column with $\boldsymbol{h}$. Then the solution to $\boldsymbol{Fg}=\boldsymbol{h}$ is $g_k=-\Delta_k/\Delta$. But $g_3^2=4g_1g_2$, that is,

$$\Delta_3^2-4\Delta_1\Delta_2=0, \tag{3.18}$$

the *explicit dispersion equation for Rayleigh waves in rhombic crystals rotating about* $\mathrm{O}x_1$. It is a polynomial of degree 12 in $X=\rho v^2$ and of degree 10 in δ^2. At $\delta=0$, $K_{23}^{(n)}=0$, so that $\Delta_1=\Delta_2=0$, while Δ_3 factorizes into the product of a quadratic in X and the cubic (3.11).

(*c*) *Rotation about* $\mathrm{O}x_2$ *(normal to the free plane)*

When the orthorhombic crystal rotates about $\mathrm{O}x_2$, the normal to the free plane, the matrices $\check{\boldsymbol{K}}^{(-2)}$ and $\check{\boldsymbol{K}}^{(2)}$ have the following structure:

$$\check{\boldsymbol{K}}^{(n)}=\begin{bmatrix}0 & \check{K}_{12}^{(n)} & 0\\ \check{K}_{12}^{(n)} & 0 & \mathrm{i}K_{23}^{(n)}\\ 0 & -\mathrm{i}K_{23}^{(n)} & 0\end{bmatrix},\quad n=-2,2, \tag{3.19}$$

where $\check{K}_{12}^{(n)}$ and $K_{23}^{(n)}$ are *real*. Explicitly,

$$s'_{11}\check{K}_{12}^{(2)}=1-[s'_{12}+s'_{11}(1+\delta^2)]X,\qquad K_{23}^{(2)}=-2\delta X, \tag{3.20}$$

and

$$\left.\begin{aligned} s'_{11}(1-s'_{66}X)D_2\check{K}_{12}^{(-2)}&=-1+[s'_{12}+(s'_{11}+s'_{55})(1+\delta^2)]X\\ &\quad-[s'_{12}(1+\delta^2)+s'_{11}(1-\delta^2)^2]s'_{55}X^2,\\ \frac{s'_{44}}{s'_{55}}s'_{11}D_2K_{23}^{(-2)}&=-2s'_{12}\delta X,\\ s'_{11}D_2&=s'_{22}-[s'_{22}(s'_{11}+s'_{55})-s'^2_{12}](1+\delta^2)X\\ &\quad+[s'_{12}(1+\delta^2)+s'_{11}(1-\delta^2)^2]s'_{55}X^2.\end{aligned}\right\} \tag{3.21}$$

Equations (3.6) written at $n=-2,2$, for $\boldsymbol{U}(0)=[1,\alpha_1+\mathrm{i}\alpha_2,\beta_1+\mathrm{i}\beta_2]^{\mathrm{T}}$, give

$$\check{K}_{12}^{(-2)}\alpha_1+K_{23}^{(-2)}(\alpha_2\beta_1-\alpha_1\beta_2)=0,\qquad \check{K}_{12}^{(2)}\alpha_1+K_{23}^{(2)}(\alpha_2\beta_1-\alpha_1\beta_2)=0. \tag{3.22}$$

Hence, either (a) $\check{K}_{12}^{(-2)}K_{23}^{(2)}-\check{K}_{12}^{(2)}K_{23}^{(-2)}=0$ or (b) $\alpha_1=\alpha_2=0$ or (c) $\alpha_1=\beta_1=0$. The first possibility is not a valid secular equation, because it is not consistent with

the non-rotating case (that is, (a) does not reduce to the cubic (3.11) at $\delta = 0$). It can be checked that in case (b) the equations (3.6) written at $n = -1, 1, 3$ lead to a secular equation which vanishes identically at $\delta = 0$, so that (b) is also dismissed. Thus (c) remains, and

$$\boldsymbol{U}(0) = [1, \mathrm{i}\alpha_2, \mathrm{i}\beta_2]^{\mathrm{T}}. \tag{3.23}$$

The directions of the vectors $\alpha_2\boldsymbol{j} + \beta_2\boldsymbol{k}$ and $\boldsymbol{i}$ are along the principal axes of the polarization ellipse at the free surface.

The matrices $\check{\boldsymbol{K}}^{(-1)}$, $\check{\boldsymbol{K}}^{(1)}$ and $\check{\boldsymbol{K}}^{(3)}$ have the following structure:

$$\check{\boldsymbol{K}}^{(n)} = \begin{bmatrix} \check{K}_{11}^{(n)} & 0 & \mathrm{i}K_{13}^{(n)} \\ 0 & \check{K}_{22}^{(n)} & \\ -\mathrm{i}K_{13}^{(n)} & 0 & \check{K}_{33}^{(n)} \end{bmatrix}, \quad n = -1, 1, 3, \tag{3.24}$$

where $\check{K}_{11}^{(n)}$, $\check{K}_{22}^{(n)}$, $\check{K}_{33}^{(n)}$ and $K_{13}^{(n)}$ are *real*. Explicitly,

$$\left.\begin{aligned} &(1 - s'_{66})X\check{K}_{11}^{(-1)} = -X, \qquad s'_{44}\check{K}_{33}^{(-1)} = 1, \qquad K_{13}^{(-1)} = 0, \\ &s'_{11}D_2\check{K}_{22}^{(-1)} = 1 - (s'_{11} + s'_{55})(1 + \delta^2) + s'_{11}s'_{55}(1 - \delta^2)^2X^2, \end{aligned}\right\} \tag{3.25}$$

where D_2 is defined in $(3.21)_3$; the components of $\check{\boldsymbol{K}}^{(1)}$ are given in $(2.11)_2$; and

$$\left.\begin{aligned} s'_{11}\check{K}_{11}^{(3)} &= \frac{s'_{66}}{s'_{11}} - 2\frac{s'_{12}}{s'_{11}} + \left[\frac{s'^2_{12}}{s'_{11}} + 2(s'_{12} - s'_{66})(1 + \delta^2)\right]X \\ &\qquad + s'_{11}[s'_{66}(1 + \delta^2)^2 + 4s'_{44}\delta^2]X^2, \\ s'_{11}\check{K}_{22}^{(3)} &= -1 + [2s'_{12} + s'_{11}(1 + \delta^2)]X + (s'_{11}s'_{22} - s'^2_{12})X^2, \\ s'_{55}\check{K}_{33}^{(3)} &= \frac{s'_{44}}{s'_{55}} - 2s'_{44}(1 + \delta^2)X + s'_{55}[s'_{44}(1 + \delta^2)^2 + 4s'_{66}\delta^2]X^2, \\ s'_{11}K_{23}^{(3)} &= 2\delta X\left[s'_{12} - s'_{66} - s'_{11}\frac{s'_{44}}{s'_{55}} + s'_{11}(s'_{44} + s'_{66})(1 + \delta^2)X\right]. \end{aligned}\right\} \tag{3.26}$$

Substitution of (3.23) into equations (3.6) written at $n = -1, 1, 3$ gives

$$\check{K}_{11}^{(n)} + \check{K}_{33}^{(n)}\beta_2^2 + \check{K}_{22}^{(n)}\alpha_2^2 - 2K_{13}^{(n)}\beta_2 = 0, \quad n = -1, 1, 3. \tag{3.27}$$

These three equations are rewritten as $F_{ik}g_k = -h_i$ with

$$\boldsymbol{F} = \begin{bmatrix} \check{K}_{33}^{(-1)} & \check{K}_{22}^{(-1)} & K_{13}^{(-1)} \\ \check{K}_{33}^{(1)} & \check{K}_{22}^{(1)} & K_{13}^{(1)} \\ \check{K}_{33}^{(3)} & \check{K}_{22}^{(3)} & K_{13}^{(3)} \end{bmatrix}, \qquad \boldsymbol{g} = \begin{bmatrix} \beta_2^2 \\ \alpha_2^2 \\ -2\beta_2 \end{bmatrix}, \qquad \boldsymbol{h} = \begin{bmatrix} \check{K}_{11}^{(-1)} \\ \check{K}_{11}^{(1)} \\ \check{K}_{11}^{(3)} \end{bmatrix}. \tag{3.28}$$

Let $\Delta = \det \boldsymbol{F}$, and Δ_k be the determinant of the matrix obtained from $\boldsymbol{F}$ by replacing the kth column with $\boldsymbol{h}$. Then the solution to $\boldsymbol{F}\boldsymbol{g} = \boldsymbol{h}$ is $g_k = -\Delta_k/\Delta$. But $g_3^2 = 4g_1$, that is,

$$\Delta_3^2 + 4\Delta_1\Delta = 0, \tag{3.29}$$

the *explicit dispersion equation for Rayleigh waves in rhombic crystals rotating about* $\mathrm{O}x_2$. It is a polynomial of degree 12 in $X = \rho v^2$ and of degree 10 in δ^2. At $\delta = 0$,

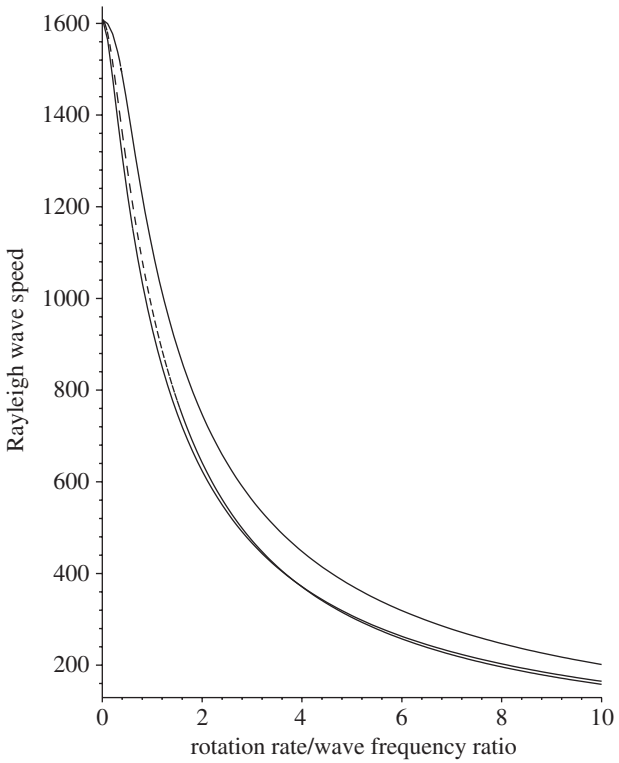

Figure 1. Rayleigh wave speeds for α-iodic acid (rhombic) rotating about $\mathrm{O}x_1$ (bottom solid curve), $\mathrm{O}x_2$ (top solid curve) or $\mathrm{O}x_3$ (dashed curve).

$K_{13}^{(n)} = 0$, so that $\Delta = \Delta_1 = 0$, while Δ_3 factorizes into the product of a quadratic in X and the cubic (3.11).

Figure 1 shows the influence of δ upon the surface wave speed for α-iodic acid (HIO_3, rhombic). The elastic stiffnesses (10^{10} N m^{-2}) are (Royer & Dieulesaint 1996) $C_{11} = 3.01$, $C_{12} = 1.61$, $C_{22} = 5.80$, $C_{44} = 1.69$, $C_{55} = 2.06$ and $C_{66} = 1.58$; the mass density is 4640 kg m^{-3}. We see that the Rayleigh wave speed is a monotone decreasing function of δ. When the crystal rotates about the x_2-axis (solid top curve), the speed is greater at a given δ than when it rotates about the x_1-axis (solid bottom curve) or the x_3-axis (dashed curve).

(*d*) *Rotation about* $\mathrm{O}x_3$

The case where the orthorhombic crystal rotates about an axis orthogonal to the direction of propagation and to the normal to the free surface has been treated elsewhere (Destrade 2004) as a special case of a monoclinic crystal. It turns out that the in-plane strain decouples from anti-plane strain, and that the displacement is of the form

$$\boldsymbol{U}(0) = [1, \mathrm{i}\alpha_2, 0]^{\mathrm{T}}, \tag{3.30}$$

where α_2 is real. Explicitly, the wave propagates at a speed which is a root of the following dispersion equation:

$$(h_1F_{23} - h_2F_{13})^2 - (F_{12}h_2 - F_{22}h_1)(F_{12}F_{23} - F_{22}F_{13}) = 0, \tag{3.31}$$

with

$$\left.\begin{aligned} F_{12} = 2s'_{12}\delta X, \qquad F_{13} = 1-(s'_{11}+s'_{66})(1+\delta^2)X + s'_{11}s'_{66}(1-\delta^2)^2X^2, \\ F_{22} = -2s'_{11}\delta X, \qquad F_{23} = s'_{11}(1+\delta^2)X, \\ h_1 = [s'_{22}(1+\delta^2) - (s'_{11}s'_{22} - s'^2_{12})(1-\delta^2)^2X]X, \qquad h_2 = 1 - s'_{11}(1+\delta^2)X, \end{aligned}\right\} \tag{3.32}$$

and the mechanical displacement at the free surface is given by (3.30), with $\alpha_2^2 = (F_{12}h_2 - F_{22}h_1)/(F_{12}F_{23} - F_{22}F_{13})$.

The dispersion equation is a polynomial of degree 6 in $X = \rho v^2$ and in δ^2. In the *isotropic* case, it reduces to $a_i\xi^i = 0$ $(i = 0, 1, \ldots, 6)$, where $\xi = \rho v^2/C_{66}$, $\alpha = C_{66}/C_{11}$, and

$$\left.\begin{aligned} a_6 &= (1-\delta^2)^4[(1+\alpha)^2(1+\delta^2)^2 - 4\alpha(1-\delta^2)^2], \\ a_5 &= -16(1-\alpha)(1+\delta^2)(1-\delta^2)^2[(1-\alpha)(1+\delta^4) + 2(1+3\alpha)\delta^2], \\ a_4 &= 16(1-\alpha)^2(1+\delta^2)^2[(7-2\alpha)(1+\delta^4) + 2(1+2\alpha)\delta^2], \\ a_3 &= -32(1-\alpha)^2(1+\delta^2)[(13-9\alpha)(1+\delta^4) + 2(7-11\alpha)\delta^2], \\ a_2 &= 64(1-\alpha)^2[(13-16\alpha+4\alpha^2)(1+\delta^2)^2 - 4\delta^2], \\ a_1 &= -256(3-2\alpha)(1-\alpha)^3(1+\delta^2), \\ a_0 &= 256(1-\alpha)^4. \end{aligned}\right\} \tag{3.33}$$

This equation corresponds to the rationalized form of the dispersion equation obtained by Clarke & Burdess (1994). In the non-rotating case, $\delta = 0$ and the sextic secular equation reduces to the (squared) cubic of Rayleigh (1885):

$$\xi^3 - 8\xi^2 + (24 - 16\alpha)\xi - 16(1-\alpha) = 0.$$

Finally, in the *incompressible isotropic* case, $\alpha = 0$ and the sextic equation factorizes into the product of the two following cubics:

$$(1+\delta^2)(1\pm\delta)^4\xi^3 - 8(1+\delta^2)(1\pm\delta)^2\xi^2 + 8(3\pm 2\delta + 3\delta^2)\xi - 16 = 0. \tag{3.34}$$

Figure 2 shows the influence of δ upon the wave speed (thick curve) for silica (SiO_2, isotropic) rotating about the normal to the directions of propagation and of attenuation. The elastic stiffnesses ($10^{10}\,\mathrm{N\,m^{-2}}$) are (Royer & Dieulesaint 1996) $C_{11} = 7.85$ and $C_{12} = 1.61$; the mass density is $2203\,\mathrm{kg\,m^{-3}}$. The thin curve corresponds to the dispersion curve obtained when the centrifugal acceleration is omitted in the equations of motion (2.6). We see that even for small δ, this term plays an important role: for instance, the speed is reduced from $3409\,\mathrm{m\,s^{-1}}$ in the non-rotating case to $3350\,\mathrm{m\,s^{-1}}$ (horizontal dashed line) for $\delta = 0.088\,52$ with the full equations of motion, and for $\delta = 0.048\,78$ without the centripetal acceleration, a relative difference of 45% for the expected value of δ! Similarly, Grigor'evskiĭ *et al.* (2000) argued that one of the bulk waves disappeared in a rotating isotropic media at $\delta = \frac{1}{2}$; in fact, as shown by Schoenberg & Censor (1973), this phenomenon occurs at 'resonance': $\delta = \Omega/\omega = 1$.

(*e*) *Remark: rotation about any direction in* $(x_3 = 0)$*-plane*

Note that the method of the polarization vector may also be carried out fully in the case where the crystal rotates about any axis in the (x_1, x_2)-plane. Then the

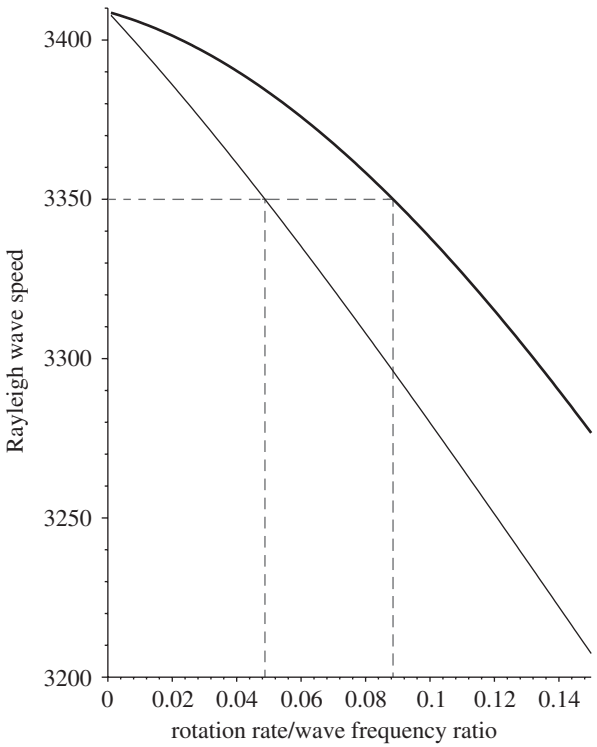

Figure 2. Rayleigh wave speeds for Silica (isotropic) rotating about Ox_3 calculated with (thick curve) and without (thin curve) the contribution of the centrifugal force.

secular equation is obtained as a nonlinear combination of determinants of 5×5 matrices, as follows.

When the unit vector $\boldsymbol{w}$ along the axis of rotation is such that $w_3 = 0$ and $w_1 w_2 \neq 0$, then the matrices $\check{\boldsymbol{K}}^{(n)}$ have the following structure:

$$\check{\boldsymbol{K}}^{(n)} = \begin{bmatrix} \check{K}_{11}^{(n)} & \check{K}_{12}^{(n)} & \mathrm{i}K_{13}^{(n)} \\ \check{K}_{12}^{(n)} & \check{K}_{22}^{(n)} & \mathrm{i}K_{23}^{(n)} \\ -\mathrm{i}K_{13}^{(n)} & -\mathrm{i}K_{23}^{(n)} & \check{K}_{33}^{(n)} \end{bmatrix}, \tag{3.35}$$

where $\check{K}_{11}^{(n)}$, $\check{K}_{22}^{(n)}$, $\check{K}_{33}^{(n)}$, $\check{K}_{12}^{(n)}$, $K_{13}^{(n)}$ and $K_{23}^{(n)}$ are *real*.

Substitution of $\boldsymbol{U}(0) = [1, \alpha_1 + \mathrm{i}\alpha_2, \beta_1 + \mathrm{i}\beta_2]^{\mathrm{T}}$ into equations (3.6) written at five different n gives

$$\left.\begin{gathered} \check{K}_{22}^{(n)} g_1 + \check{K}_{33}^{(n)} g_2 + \check{K}_{12}^{(n)} g_3 + K_{13}^{(n)} g_4 + K_{23}^{(n)} g_5 = -\check{K}_{11}^{(n)}, \\ g_1 = \alpha_1^2 + \alpha_2^2, \quad g_2 = \beta_1^2 + \beta_2^2, \quad g_3 = 2\alpha_1, \quad g_4 = -2\beta_2^2, \quad g_5 = 2(\alpha_2\beta_1 - \alpha_1\beta_2). \end{gathered}\right\} \tag{3.36}$$

The non-homogeneous linear system of five equations is solved for the five unknowns g_i. But these quantities are related through

$$g_1^2 g_4 + g_3^2 g_2 + g_5^2 - g_1 g_3 g_5 - 4 g_2 g_4 = 0, \tag{3.37}$$

the *dispersion equation for Rayleigh waves in rhombic crystals rotating about any axis in the* (x_1, x_2)*-plane*.

Explicitly, the components of the $\check{\boldsymbol{K}}^{(n)}$ matrices are, however, too long to reproduce here. Also, the corresponding set-ups are unlikely to be of experimental relevance.

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