MA343 Groups I
Sample Class Test 2007

Answer four questions.

1. Let $a = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$. Prove that $G = \langle a \rangle$ is a finite cyclic group (with respect to matrix multiplication). What is the order of G?

2. Show that the set $\mathbb{Z}_8^*$ of congruence classes mod 8 coprime to 8 forms a group with respect to multiplication. What is the order of $\mathbb{Z}_8^*$? Is the group cyclic?

3. Let $C_{20} = \langle a \mid a^{20} = 1 \rangle$.
(i) Give an example of a subgroup of order 4 in C_{20}.
(ii) Prove that $a^{-5} = a^{15}$.
(iii) Does C_{20} have a subgroup of order 9?
(iv) What is the order of the subgroup $H = \langle a^5 \rangle$? List all elements of H. Find $|G : H|$.

4. Let $G = \mathrm{Sym}(3)$ and let $H = \langle (2, 1, 3) \rangle$. List all left cosets of H in G.

5. Let $G = \{(a, 2a + 1) \mid a \in \mathbb{Z}\}$. Is G a group with respect to the usual vector addition?

6. Let $H = \langle (4, 3, 1) \rangle \leq \mathrm{Alt}(4)$. What is the order of H, and what is the index of H?