Mathematics in Wonderland

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Alice in Wonderland

Alice's Adventures in Wonderland

1865 Novel

Lewis Carroll

Huge influence on popular culture

Memorable characters and scenes

Were not all there from the start

Where did they come from?

Where is the maths?
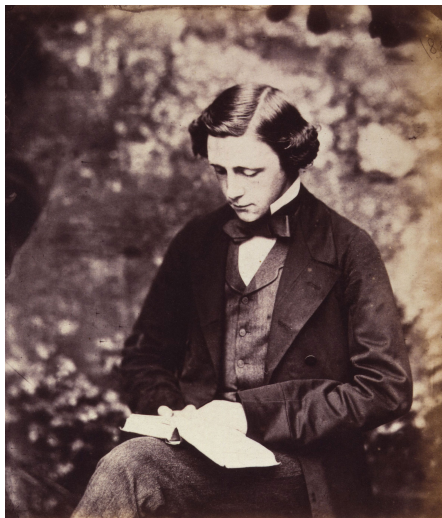
Lewis Carroll aka Charles Lutwidge Dodgson

Figure: Lewis Carroll

- Mathematician
  - ▶ 1st class honours in maths at Oxford
  - ▶ Taught at Christ Church for 26 years
- 1862 boat trip on the Thames
  - ▶ Reverend Robinson Duckworth, Lorina (13), Alice (10), Edith (8)
  - ▶ Inspired by places in Oxford
  - ▶ Alice asked him to write the stories down
- 1864 handwritten illustrated manuscript *Alice's Adventures Under Ground*
  - ▶ Cheshire Cat, the trial, the Duchess's baby, and the Mad Hatter's tea party not included

A Traditionalist?

Figure: First English translation of Euclid's Elements (1570)

- Considered a very conservative mathematician
  - ▶ Axiomatic approach of Euclid's Elements
  - ▶ Physical meaning and geometry
- Tutor rather than a research mathematician
  - ▶ Disliked new abstract work, questioned rigour
  - ▶ Only for other mathematicians, not students
- Publications
  - ▶ A Syllabus of Plane Algebraic Geometry (1860)
  - ▶ The Fifth Book of Euclid Treated Algebraically (1858 and 1868)
  - ▶ *Alices Adventures in Wonderland (1865)*
  - ▶ An Elementary Treatise on Determinants, With Their Application to Simultaneous Linear Equations and Algebraic Equations (1867)
  - ▶ *Through the Looking Glass (1872)*
  - ▶ Euclid and his Modern Rivals (1879)
  - ▶ Symbolic Logic Part I (1896)
  - ▶ Symbolic Logic Part II (published posthumously)

Traditionalist Mathematicians

Even negative numbers were once controversial, in fact in Europe they were controversial right up through the Victorian era.

Negative numbers *"... darken the very whole doctrines of the equations and make dark of the things which are in their nature excessively obvious and simple"*. Francis Maseres, British Mathematician, 1758.

Used since

- 200 BC in China
- 600 AD in India
- 800 AD in the Middle East
- not until the 2400s in Europe

The Greeks dealt mostly with Geometry

- positive lengths, areas and volumes

Arithmetic and algebra came from the Arabs. First brought over to Europe by Leonardo of Pisa, aka Fibonacci in the 1200s. His book, Liber Abaci, didn't contain any negative numbers, despite dealing with money.

Nonsense Words and New Mathematics
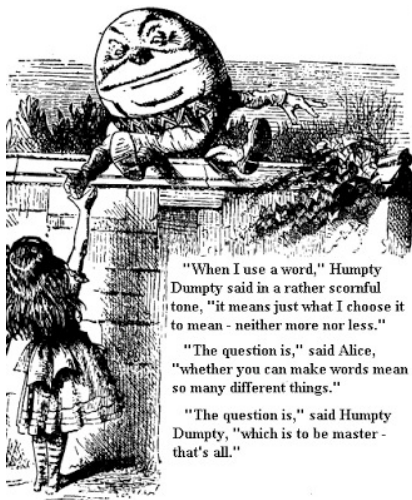
"When I use a word," Humpty Dumpty said in a rather scornful tone, "it means just what I choose it to mean - neither more nor less."

"The question is," said Alice, "whether you can make words mean so many different things."

"The question is," said Humpty Dumpty, "which is to be master - that's all."

Figure: Humpty Dumpty in *Through the Looking Glass*

- Humpty Dumpty
  - ▶ Dialogue full of nonsense words
  - ▶ Spoof on modern abstract mathematics
    - ★ giving very specific technical meanings to such everyday words as set, group, ring, field, etc

The Base System

Let me see: four times five is twelve, and four times six is thirteen, and four times seven is – oh dear! I shall never get to twenty at that rate! However, the Multiplication Table don't signify....

Figure: Alice Multiplying in *Alice's Adventures in Wonderland*

Symbolic Algebra and Euclidean Geometry

Figure: The Caterpillar in *Alice's Adventures in Wonderland*

Figure: Alice eats the mushroom in *Alice's Adventures in Wonderland*

- Caterpillar enters, smoking a *hookah* pipe, and shows Alice a mushroom that can restore her to her proper size.
  - ▶ One side of the mushroom stretches her neck, while another shrinks her torso.
  - ▶ Need to eat exactly the right balance.
  - ▶ Dodgson's view of the absurdity of symbolic algebra.
- Arabic term for algebra, widely used in Dodgson's time, was *al jebr e al mokabala* or restoration and reduction.
- *"Keep your temper"*
  - ▶ Temper has another meaning - *the proportion in which qualities are mingled.* Caterpillar telling Alice to keep her body in proportion - no matter what her size.
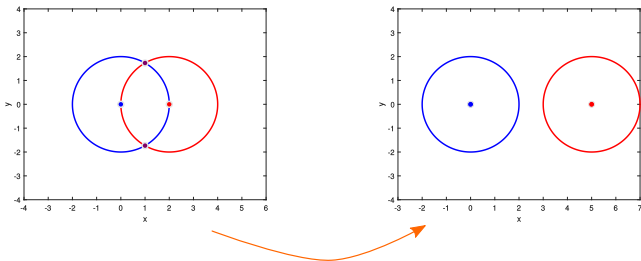 - ▶ Act Like a Euclidean geometer.

Projective Geometry

Figure: The Principle of Continuity: Circles intersect at $(1, \pm 3)$ and $(5/2, \pm 3/2i)$ and after transformation.

Figure: Alice and the pig-baby.

"Let a figure be conceived to undergo a certain continuous variation, and let some general property concerning it be granted as true, so long as the variation is confined within certain limits; then the same property will belong to all the successive states of the figure"
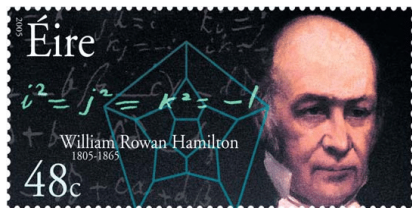Jean-Victor Poncelet - French mathematician
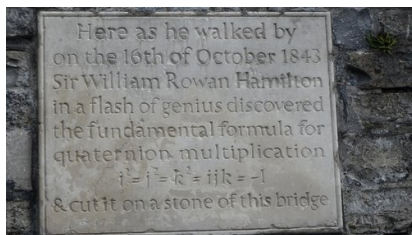
Quaternions

Figure: William Rowan Hamilton.

Figure: Broom Bridge, Dublin.

- Struggled to find an algebraic way of describing rotations in 3D.
  - All known algebraic systems were commutative.
- Flash of inspiration while walking along the Royal Canal in Dublin.
  - 16 October 1843 on the way to an RIA Council Meeting

"And here there dawned on me the notion that we must admit, in some sense, a fourth dimension of space for the purpose of calculating with triples . . . An electric circuit seemed to close, and a spark flashed forth."

$$i^2 = j^2 = k^2 = ijk = -1.$$

Quaternions

Figure: The Mad Hatter's t-party

"It seemed (and still seems) to me natural to connect this extra-spatial unit with the conception of time." Hamilton in a preface to his Lectures on Quaternions of 1853

- Alice with three strange characters: the Hatter, the March Hare and the Dormouse.
- Time fallen out with the Hatter - absent.

- Without Time they are stuck at the tea table, constantly moving round and round, just like the quaternions without a 4th term.
- *"Why is a raven like a writing desk?"* targets the theory of pure time. In the realm of pure time, Hamilton claimed, cause and effect are no longer linked, the madness of the Hatter's question reflects this.
- Alices ensuing attempt to solve the riddle pokes fun at the quaternion's non-commutative multiplication. Hare tells her to *"say what she mean"*, she replies that she does, *"at least I mean what I say - that's the same thing"*. Hatter: *"you might just as well say that 'I see what I eat' is the same thing as I eat what I see'!"*

Summary

Figure: Salvador Dali's Mad Tea Party (1969)

- Original nursery tale, Alice's Adventures Under Ground didn't have the characteristic nonsense.
- Most witty when poking fun at something.
  - He wrote two funny pamphlets, in the style of mathematical proofs, ridiculing changes at the University of Oxford.
  - Other stories he wrote besides the Alice books were dull and moralistic.

Without Dodgson's fierce satire would Alice's Adventures in Wonderland have become famous? Would Lewis Carroll be remembered as the unrivalled master of nonsense fiction?

References & Links

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