Control of a model of competition between two animal species

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The Model

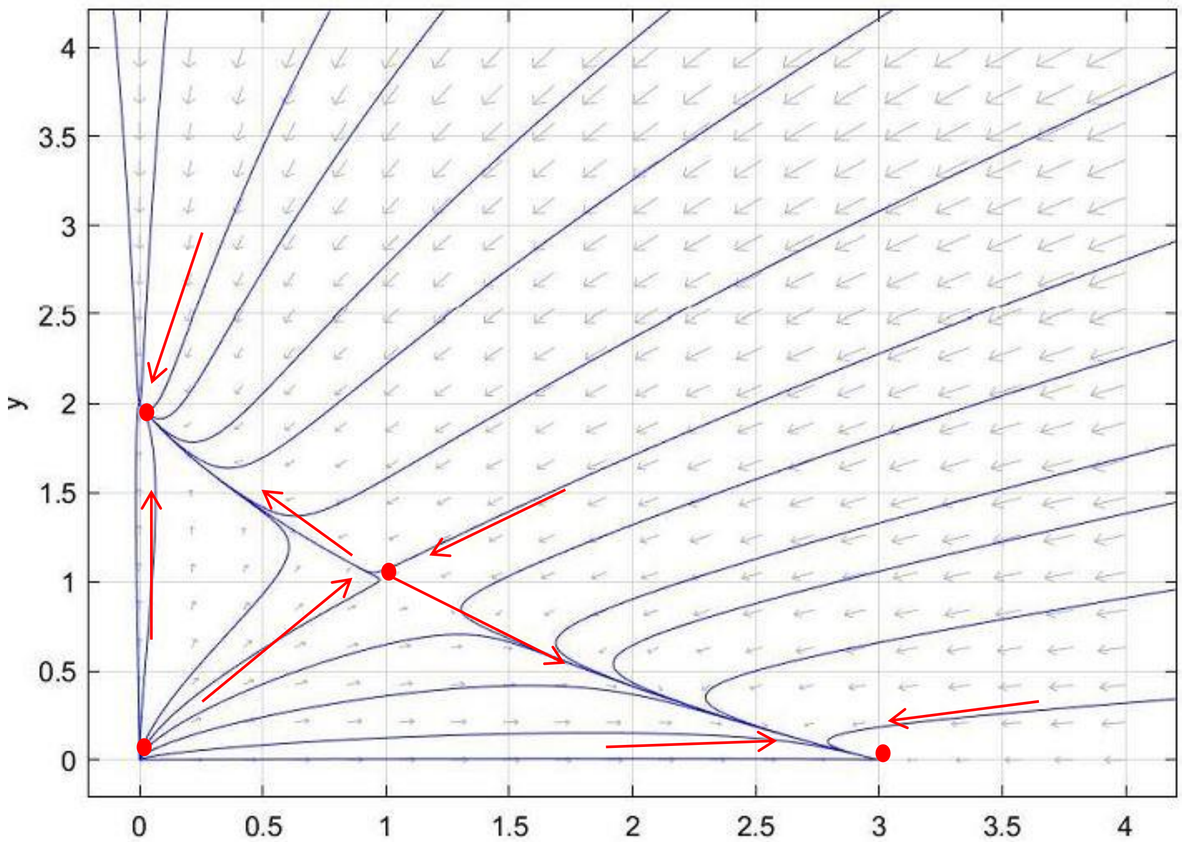
$$\frac{dx_1}{dt} = x_1(a_1 - b_1x_1 - c_1x_2)$$
$$\frac{dx_2}{dt} = x_2(a_2 - b_2x_1 - c_2x_2)$$

$$x, y \geq 0$$

$\boldsymbol{x_1}$(t) = population of species1 (i.e. rabbits)
$\boldsymbol{x_2}$(t) = population of species 2 (i.e. Sheep)

- Competition for the same food supply and the amount available is limited.
- Each species would grow to its carrying capacity in the absence of the other.
- When both species coexist they start fighting for food.

Phase portrait



- *Principle of competitive exclusion:* two species competing for the same limited food typically cannot coexist
- Basins and their boundaries partition the phase plane into regions of different long-term behavior

First control strategy

Aim: to incentive the coexistence of species

- Proportional error controller:

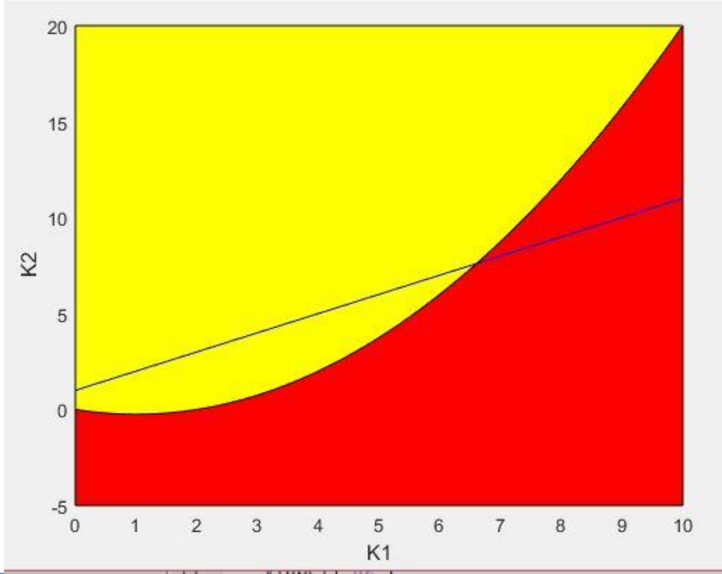
$$u = K_1(x_{1ref} - x_1) + K_2(x_{2ref} - x_2)$$

Varying K_1 and K_2, the dynamics of the closed loop system change :

If $K_2 = K_1 + 1$ (1;1) is unstable (blue line)

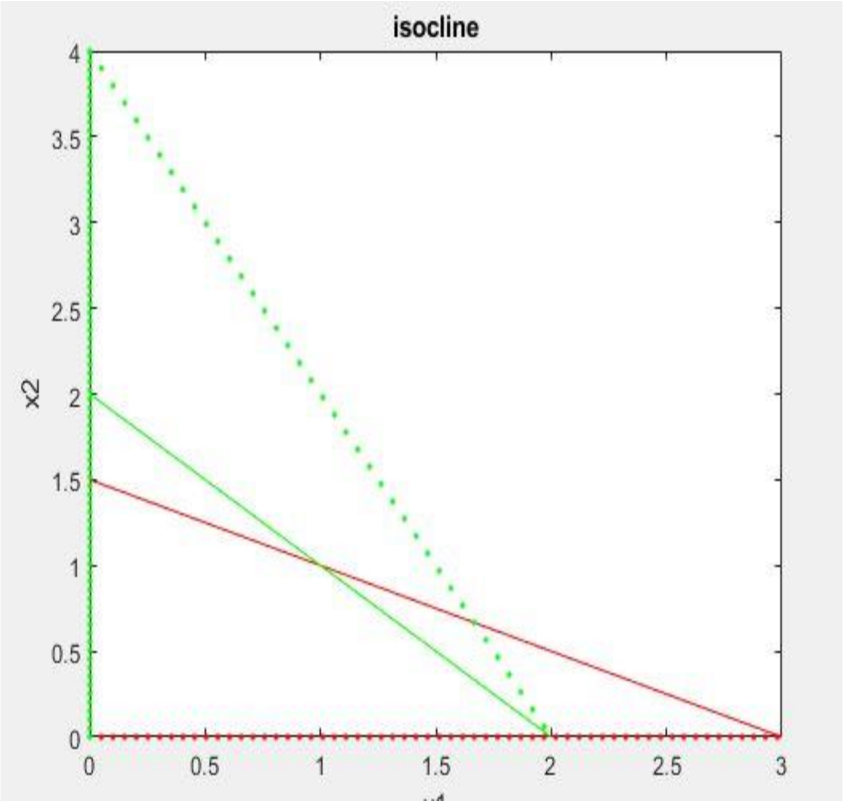
If $K_2 < \frac{{K_1}^2}{4} - \frac{K_1}{2}$ (1;1) is a stable focus (red)

If $K_2 \geq \frac{{K_1}^2}{4} - \frac{K_1}{2}$ (1;1) is a stable node (yellow)



Second Control Strategy

Aim: to make the basins of attraction bigger or smaller



- Using a different value of c_1 and c_2 the equilibrium points change.
- Control input:

$$\dot{x_1} = x_1(a_1 - b_1 x_1 - (c_1 + u)x_2)$$
$$\dot{x_2} = x_2(a_2 - b_2 x_1 - (c_2 + u)x_2)$$

Control on competition.

Note: varying the equilibrium points we have to check the stability doesn't change.

Future Plans

- Finish Thesis work
- Finish last exams
- Graduate