Simulating Filippov Systems

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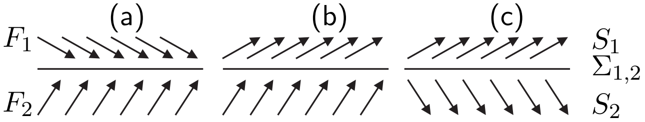
Filippov systems with **one** switching surface

In [PP. and Kuznetsov 2008] we developed a simple simulation routine for Filippov systems of the type

$$\dot{x} = \begin{cases} F_1(x), & x \in S_1, \\ F_2(x), & x \in S_2, \\ F_{1,2}(x), & x \in \Sigma_{1,2}. \end{cases}$$

with $\Sigma_{1,2} := \{x \in S \mid H(x) = 0\}$. Depending of F_1 and F_2 the *discontinuity / Filippov / sliding / switching surface* $\Sigma_{1,2}$ can be **attractive**, **repelling** or **switching**.



To find $F_{1,2}(x)$ *Utkin's eqvivalent method* states that

$$F_{1,2}(x) = \frac{1+\lambda}{2} F_1(x) + \frac{1+\lambda}{2} F_2(x), \quad \lambda \in [-1, 1].$$

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However, I would also like to be able to consider non-convex methods such that

$$\dot{x} = \frac{1+\lambda}{2}F_1(x) + \frac{1-\lambda}{2}F_2(x) + (\lambda^2 - 1)G(x),$$

for some function G and where the λ concept has been extended to

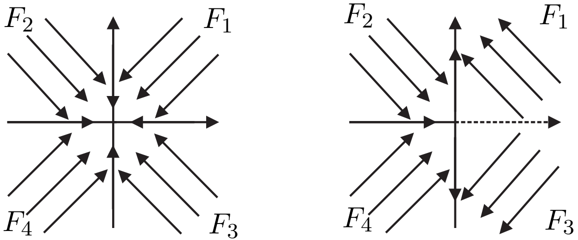
$$\begin{aligned} \lambda &= \text{sign}(H(x)) \quad &\text{for} \quad &H(x) \neq 0, \\ \lambda &\in (-1, 1) \quad &\text{for} \quad &H(x) = 0 \end{aligned}$$

and where the term $(\lambda^2 - 1)G(x)$ is called **hidden** since it vanishes when $x \notin \Sigma$.

Why?

Filippov systems with **two** switching surfaces

Consider a system with two switching surfaces and four vector fields.



If we use Utkin's equivalent method, in the general case, we will have **four unknowns** and only **three equations** to resolve the dynamics on the manifold given by the crossing of the two surfaces.

Switching layer and λ dynamics

Simple example. Consider

$$(\dot{x}_1, \dot{x}_2) = (2 + \lambda, 1) + 2(\lambda^2 - 1, 0)$$

with $H(x_1, x_2) = x_1$ and let us consider what happens on $\dot{H} = H = 0$. We get

$$\dot{x}_1 = 0 \ \rightarrow \ 2 + \lambda + 2\lambda^2 - 2 = 0 \ \rightarrow \ \lambda = -1/2, 0,$$

i.e. **two solutions**. Which one is the correct one? The trick is to introduce a **switching layer** (of width ε) with λ dynamics

$$\varepsilon\dot{\lambda} = \nabla H \bullet F, \quad \dot{\lambda} \sim 1/\varepsilon.$$

Thus, on $H = x_1 = 0$ we have $\varepsilon\dot{\lambda} = 2 + \lambda + 2(\lambda^2 - 1)$ and since

$$\frac{d\dot{\lambda}}{d\lambda} = 1 + 4\lambda = -1 \text{ (for } \lambda = -1/2), \ 1 \text{ (for } \lambda = 0)$$

the solution $\lambda = -1/2$ is the stable solution the flow follows with $\dot{x}_1 = 0, \ \dot{x}_2 = 1$.

Plans

Supervision: Setting up projects, Paper writing, Thesis reading

Book: Starting a book project with Raghav.

Papers: A number of papers in the pipeline with Raghav, John D, David C & Arne N, Joanna J etc.

Programming: Filippov solver for systems with nonlinear terms.

Travel: India 16 Dec 18 - 8 Mar 19, USA around 20-30 Mar 19, Napoli Apr 19, ...

2-0?