A Characterisation of Clique Graphs

Cian O'Brien
Rachel Quinlan and Kevin Jennings

Postgraduate Modelling Research Group
National University of Ireland, Galway

c.obrien40@nuigalway.ie

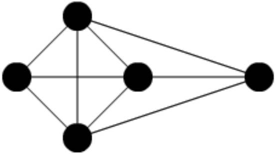
February 9th, 2018

What is a Clique?

A *clique* C of of a graph G is a subgraph of G which is complete and is contained in no larger complete subgraph of G.

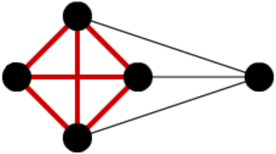
What is a Clique?

A *clique* C of of a graph G is a subgraph of G which is complete and is contained in no larger complete subgraph of G.



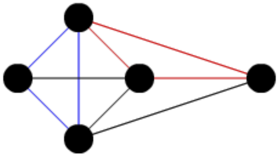
What is a Clique?

A *clique* C of of a graph G is a subgraph of G which is complete and is contained in no larger complete subgraph of G.



What is a Clique?

A *clique* C of of a graph G is a subgraph of G which is complete and is contained in no larger complete subgraph of G.



What is a Clique Graph?

A *clique graph* of G is a graph F and a $1 - 1$ onto mapping α from $V(F)$ to the set of cliques of G which preserves incidence.

What is a Clique Graph?

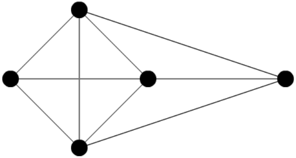
A *clique graph* of G is a graph F and a $1-1$ onto mapping α from $V(F)$ to the set of cliques of G which preserves incidence.

i.e. $x, y \in V(F)$ are adjacent iff $V(\alpha(x)) \cap V(\alpha(y)) \neq \emptyset$

What is a Clique Graph?

A *clique graph* of G is a graph F and a $1-1$ onto mapping α from $V(F)$ to the set of cliques of G which preserves incidence.

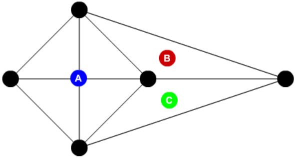
i.e. $x, y \in V(F)$ are adjacent iff $V(\alpha(x)) \cap V(\alpha(y)) \neq \emptyset$



What is a Clique Graph?

A *clique graph* of G is a graph F and a $1-1$ onto mapping α from $V(F)$ to the set of cliques of G which preserves incidence.

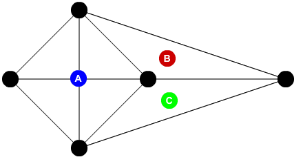
i.e. $x, y \in V(F)$ are adjacent iff $V(\alpha(x)) \cap V(\alpha(y)) \neq \emptyset$

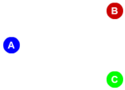


What is a Clique Graph?

A *clique graph* of G is a graph F and a $1-1$ onto mapping α from $V(F)$ to the set of cliques of G which preserves incidence.

i.e. $x, y \in V(F)$ are adjacent iff $V(\alpha(x)) \cap V(\alpha(y)) \neq \emptyset$

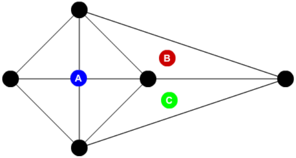


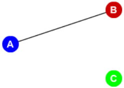


What is a Clique Graph?

A *clique graph* of G is a graph F and a $1-1$ onto mapping α from $V(F)$ to the set of cliques of G which preserves incidence.

i.e. $x, y \in V(F)$ are adjacent iff $V(\alpha(x)) \cap V(\alpha(y)) \neq \emptyset$

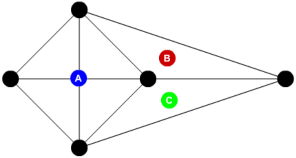


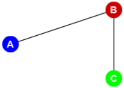


What is a Clique Graph?

A *clique graph* of G is a graph F and a $1-1$ onto mapping α from $V(F)$ to the set of cliques of G which preserves incidence.

i.e. $x, y \in V(F)$ are adjacent iff $V(\alpha(x)) \cap V(\alpha(y)) \neq \emptyset$

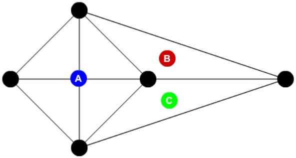


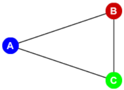


What is a Clique Graph?

A *clique graph* of G is a graph F and a $1-1$ onto mapping α from $V(F)$ to the set of cliques of G which preserves incidence.

i.e. $x, y \in V(F)$ are adjacent iff $V(\alpha(x)) \cap V(\alpha(y)) \neq \emptyset$





Identifying Clique Graphs

Question: How can we tell if a given graph H is the clique graph of some graph G, and if it is, how do we find one such graph?

Identifying Clique Graphs

Question: How can we tell if a given graph H is the clique graph of some graph G, and if it is, how do we find one such graph?

- Label the n cliques $k_1, k_2, ..., k_n$.

Identifying Clique Graphs

Question: How can we tell if a given graph H is the clique graph of some graph G, and if it is, how do we find one such graph?

- Label the n cliques $k_1, k_2, ..., k_n$.
- Let vertex $v \in V_s$, where s is an element of the set of subsets of $\{1, 2, ..., n\}$, if s is the indices for the largest subset of cliques that all contain v.

Identifying Clique Graphs

Question: How can we tell if a given graph H is the clique graph of some graph G, and if it is, how do we find one such graph?

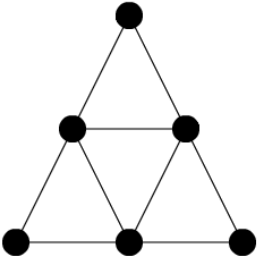
- Label the n cliques $k_1, k_2, ..., k_n$.
- Let vertex $v \in V_s$, where s is an element of the set of subsets of $\{1, 2, ..., n\}$, if s is the indices for the largest subset of cliques that all contain v.
- G has one vertex corresponding to each vertex in H, as well as one vertex corresponding to each clique in H. Clique vertices are connected to one another if both of their indices occur in a non-empty V_i. Connect an original vertex to all the clique vertices corresponding to the set V_s it was assigned to.

Example

Are there graphs that are not the clique graph of any graph?

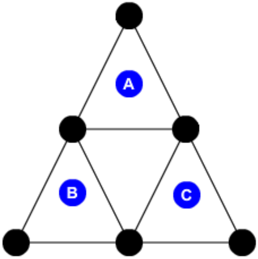
Example

Are there graphs that are not the clique graph of any graph?



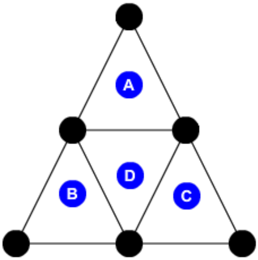
Example

Are there graphs that are not the clique graph of any graph?



Example

Are there graphs that are not the clique graph of any graph?



The Characterisation

Let K be a collection of complete subgraphs of a graph H. We say K has property I if, whenever $L_1, L_2, ..., L_p$ are in K and $L_i \cap L_j \neq 0$ for all i, j, then the total intersection is non empty.

The Characterisation

Let K be a collection of complete subgraphs of a graph H. We say K has property I if, whenever $L_1, L_2, ..., L_p$ are in K and $L_i \cap L_j \neq 0$ for all i, j, then the total intersection is non empty.

Theorem: A graph H is a clique graph iff there is a collection K of complete subgraphs of H which satisfies the following two properties:

The Characterisation

Let K be a collection of complete subgraphs of a graph H. We say K has property I if, whenever $L_1, L_2, ..., L_p$ are in K and $L_i \cap L_j \neq 0$ for all i, j, then the total intersection is non empty.

Theorem: A graph H is a clique graph iff there is a collection K of complete subgraphs of H which satisfies the following two properties:

- K covers all the edges of H.

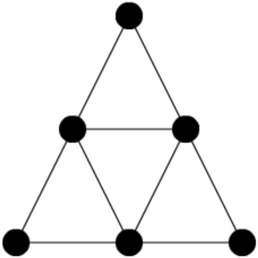
The Characterisation

Let K be a collection of complete subgraphs of a graph H. We say K has property I if, whenever $L_1, L_2, ..., L_p$ are in K and $L_i \cap L_j \neq 0$ for all i, j, then the total intersection is non empty.

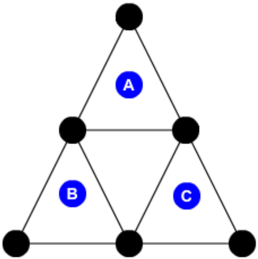
Theorem: A graph H is a clique graph iff there is a collection K of complete subgraphs of H which satisfies the following two properties:

- K covers all the edges of H.
- K satisfies property I.

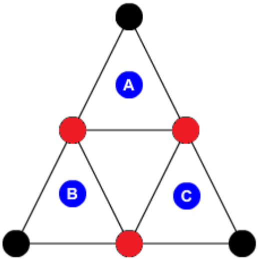
Example



Example



Example



Ronald C. Hamelink, *A Partial Characterization of Clique Graphs*, Journal of Combinatorial Theory 5, 192-197 1968

Fred S. Roberts and Joel H. Spencer, *A Characterization of Clique Graphs*, Journal of Combinatorial Theory 10, 102-108 1971