Nonsmooth Dynamical Systems with Noise: An Introduction

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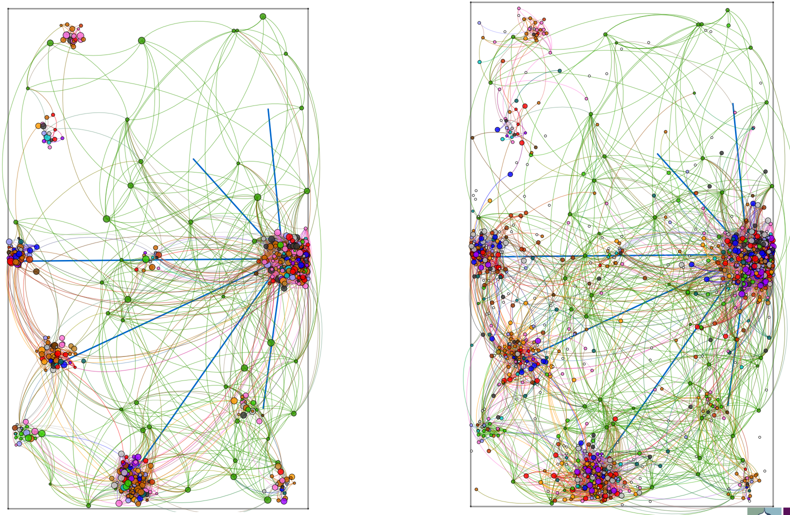
Something Completely Different - Economic Development



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Introduction

Historically mathematicians have made widespread use of smooth, deterministic mathematical models to describe real-world phenomena. These models present a simplified view of the world where

1. The evolution of systems is always smooth and exhibits no interruptions such as impacts, switches, slides or jumps.
2. The future of any system is completely determined by its present state.

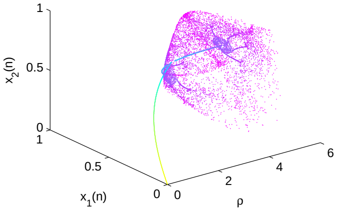

Figure: Bifurcation diagram for a smooth dynamical system on a network with two nodes

Introduction

However, when modelling many real-world systems one or both of these simplifications may not hold.

1. A level of randomness or noise is ubiquitous in real-world systems.
2. Many real-world systems behave in a nonsmooth manner:
   - Mechanical systems through impacts or friction
   - Electrical systems through switches
   - More complex systems such as the world's climate and financial systems have also been modelled using nonsmooth models.

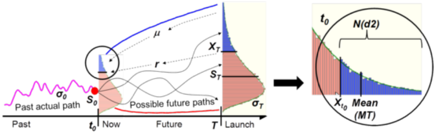

Figure: Models for option valuation.

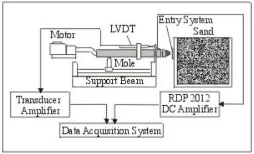

Figure: Schematic of a vibro-impacting system.

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Introduction

Both noise and nonsmoothness have been shown to be the drivers of significant changes in qualitative behaviour.

- Nonsmooth systems - qualitative changes in the behavior of the system under parameter variation that do not occur in the smooth setting.
- Adding noise to *(smooth)* systems - does more than just blur the outcome of the system in the absence of noise.

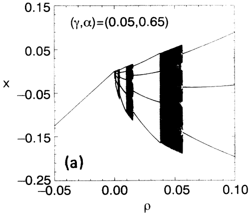

Figure: Grazing bifurcation diagram.

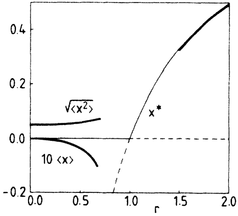

Figure: Additive noise destroys the transcritical bifurcation at r= 1 in the logistic map.

Central Research Questions

1. How can we best include noise in a nonsmooth dynamical system?
2. What are the potential outcomes of including noise in nonsmooth dynamical systems and in particular how are DIBs affected by the addition of noise?
3. Can we classify how noise enters and affects different types of nonsmooth dynamical systems?
4. What numerical methods are suitable for simulating nonsmooth, noisy dynamical systems?

Why nonsmooth?

Nonsmooth systems $\sim$ systems whose solutions are not everywhere differentiable, and may even possess discontinuities.

- Well-developed approaches to dynamical systems - typically rely on the system evolution being defined by a smooth function of its arguments.
- This excludes many systems that arise in practice.
- Can be argued all physical systems are smooth in reality.
- Timescales over which transitions such as impacts occur are so small compared to the overall dynamics that the appropriate global model is nonsmooth on a macroscopic timescale.

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Example: DC/DC Buck Converter

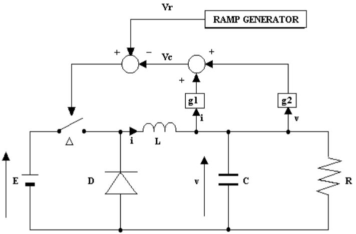

Figure: Schematic of DC/DC Buck Converter.

- Several coexisting attractors can be detected.
- Abrupt transition from periodic orbit to chaos.
- Initial attempts to account for the experimental observations using the existing theory of bifurcations in smooth dynamical systems failed
- Transitions observed are due to the discontinuous nature of the circuit.

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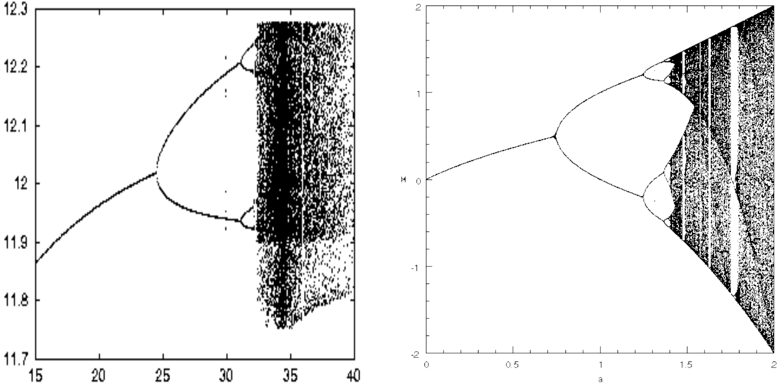

Figure: Comparison of bifurcation diagrams for a Buck Converter (on the left) and a smooth dynamical system (on the right).

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References

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