3d flocking model

Richie Burke

February 17, 2017

Motivation

The analysis and modelling of flocks/swarms has found wide application in areas such as

Motivation

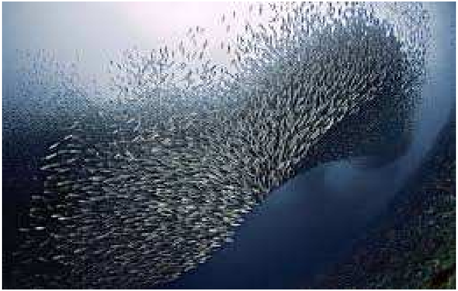
The analysis and modelling of flocks/swarms has found wide application in areas such as

- biology/ecology
- civil engineering/crowd control
- fisheries
- robotics/unmanned aerial vehicles

Motivation

The analysis and modelling of flocks/swarms has found wide application in areas such as

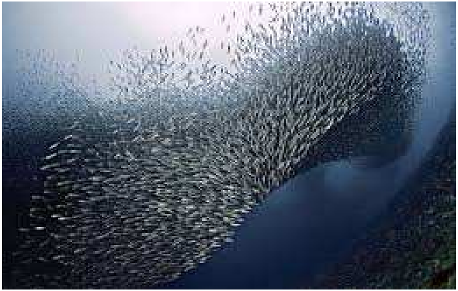
- biology/ecology
- civil engineering/crowd control
- fisheries
- robotics/unmanned aerial vehicles



Motivation

The analysis and modelling of flocks/swarms has found wide application in areas such as

- biology/ecology
- civil engineering/crowd control
- fisheries
- robotics/unmanned aerial vehicles



We shall investigate swarming models from the perspective of hybrid multi-agent control/consensus.

Consensus

Broadly speaking, *consensus* occurs when the many agents adjust their positions/velocities in relation to one another and reach some "agreement" such as a formation in space.

Consensus

Broadly speaking, *consensus* occurs when the many agents adjust their positions/velocities in relation to one another and reach some "agreement" such as a formation in space.

Toy model

We construct a model of N flocking agents by assigning each agent in the system both a

- state value $\mathbf{s}_i = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$
- gain $\sigma_{i,j}$

which correspond, respectively, to position in space and communicative strengths.

Differential equations

The state and gain evolutions are governed by a system of coupled differential equations. The general form being:

Differential equations

The state and gain evolutions are governed by a system of coupled differential equations. The general form being:

$$\dot{\mathbf{s}}_i = \underbrace{\mathbf{f}(\mathbf{s}_i)}_{\text{independent kinematics}} + \underbrace{\sum_{j \neq i} \zeta(\sigma_{i,j}, \mathbf{s}_i, \mathbf{s}_j, \Omega_i)}_{\text{local influence}} + \underbrace{\eta(\mathbf{s}_i, \mathbf{s}_j)}_{\text{global influence}} \tag{1}$$

Differential equations

The state and gain evolutions are governed by a system of coupled differential equations. The general form being:

$$\dot{\mathbf{s}}_i = \underbrace{\mathbf{f}(\mathbf{s}_i)}_{\text{independent kinematics}} + \underbrace{\sum_{j \neq i} \zeta(\sigma_{i,j}, \mathbf{s}_i, \mathbf{s}_j, \Omega_i)}_{\text{local influence}} + \underbrace{\eta(\mathbf{s}_i, \mathbf{s}_j)}_{\text{global influence}} \qquad (1)$$

$$\dot{\sigma}_{i,j} = \xi(\mathbf{s}_i, \mathbf{s}_j, \Omega_i) \qquad (2)$$

where Ω_i is the local neighbourhood of agent i containing n members and ξ is some function of the respective states.

Particular form

We aim to tailor equations (1) and (2) to effect flocking in a multi agent model.

Particular form

We aim to tailor equations (1) and (2) to effect flocking in a multi agent model.

$$\Delta_{i,j} = \mathbf{s}_i - \mathbf{s}_j = \begin{pmatrix} x_i - x_j \\ y_i - y_j \\ z_i - z_j \end{pmatrix}$$

Particular form

We aim to tailor equations (1) and (2) to effect flocking in a multi agent model.

$$\Delta_{i,j} = \mathbf{s}_i - \mathbf{s}_j = \begin{pmatrix} x_i - x_j \\ y_i - y_j \\ z_i - z_j \end{pmatrix}$$

We assign an orientation vector for each pair of states

$$\mathbf{\Theta}_{i,j} = \begin{pmatrix} \sin\phi_{i,j}\cos\theta_{i,j} \\ \sin\phi_{i,j}\sin\theta_{i,j} \\ \cos\theta_{i,j} \end{pmatrix}$$

A desired radius d is prescribed to define an *objective function* $\psi_{i,j} = \|\Delta_{i,j}\| - d^2$

A desired radius d is prescribed to define an *objective function* $\psi_{i,j} = \|\Delta_{i,j}\| - d^2$

$$w_{i,j} = \begin{cases} \sigma_{i,j} & \text{if } \psi_{i,j} \geq 0, \\ b & \text{otherwise.} \end{cases}$$

A desired radius d is prescribed to define an *objective function* $\psi_{i,j} = \|\Delta_{i,j}\| - d^2$

$$w_{i,j} = \begin{cases} \sigma_{i,j} & \text{if } \psi_{i,j} \geq 0, \\ b & \text{otherwise.} \end{cases}$$

The state evolutions are now governed by

$$\dot{\mathbf{s}}_i = \sum_{j \neq i} w_{i,j} \psi_{i,j} \mathbf{\Theta}_{i,j}$$

A desired radius d is prescribed to define an *objective function* $\psi_{i,j} = \|\Delta_{i,j}\| - d^2$

$$w_{i,j} = \begin{cases} \sigma_{i,j} & \text{if } \psi_{i,j} \geq 0, \\ b & \text{otherwise.} \end{cases}$$

The state evolutions are now governed by

$$\dot{\mathbf{s}}_i = \sum_{j \neq i} w_{i,j} \psi_{i,j} \mathbf{\Theta}_{i,j}$$

Whereas the gains evolve according to

$$\dot{\sigma}_{i,j} = \begin{cases} \alpha & \text{if } \|\Delta_{i,j}\| \in \Omega_i, \ \sigma_{i,j} < \tau \\ -\beta & \text{if } \|\Delta_{i,j}\| \notin \Omega_i, \ \sigma_{i,j} > 0 \\ 0 & \text{otherwise.} \end{cases}$$

References

- “Flocks, Herds, and Schools: A Distributed Behavioral Model, in Computer Graphics”, C.W. Reynolds, 21(4) (SIGGRAPH ’87 Conference Proceedings) pp. 25-34 (1987).
- “Pattern formation and functionality in swarm models”, E.M. Rauch, M.M . Millonas and D.R. Chialvo, Physics Letters A 207, pp. 185-193 (1995).
- “Interaction ruling animal collective behavior depends on topological rather than metric distance: Evidence from a field study”, M. Ballerini, N. Cabibbo, R. Candelier, A. Cavagna, E. Cisbani, I. Giardina, V. Lecomte, A. Orlandi, G. Parisi, A. Procaccini, M. Viale and V. Zdravkovic, PNAS vol. 105 no. 4, pp. 1232-1237 (2008).
- “New tools for characterizing swarming systems: A comparison of minimal models”, C. Huepe and M. Aldana, Physica A 387, pp. 28092822 (2008).