Fnding Flocks

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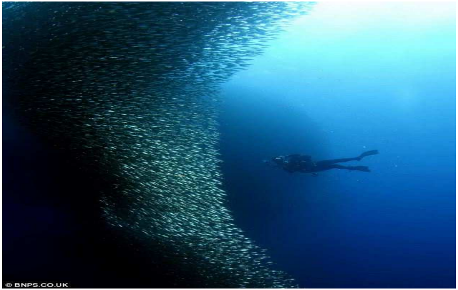

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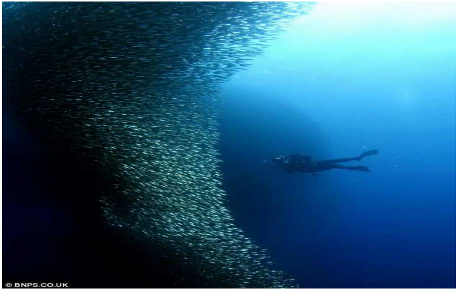

We shall investigate swarming models from the perspective of hybrid multi-agent control/consensus.

Consensus

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Flocking models

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Various metrics have been proposed to measure substructures in networks, ideas like modularity, cliques, cycles and reachability spaces have been tailored by graph theorists and computer scientists to search through large data structures and evince connected components in efficient ways.

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$$\dot{\sigma}_{i,j} = \xi(\mathbf{s}_i, \mathbf{s}_j, \Omega_i) \tag{2}$$

where Ω_i is the local neighbourhood of agent i containing n members and ξ is some function of the respective states.

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Hereafter we shall define the norm $\|\cdot\|$ as the square of the euclidean. To be explicit

$$\|\Delta_{i,j}\| = (x_i - x_j)^2 + (y_i - y_j)^2$$

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Particular form

The meso-attraction/micro-repulsion for the state kinematics is included via the following switch function

$$z_{i,j} = \begin{cases} \sigma_{i,j} & \text{if } \psi_{i,j} \geq 0, \\ b & \text{otherwise.} \end{cases}$$

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Now neglecting the independent kinematics of each agent $\mathbf{f}(\mathbf{s}_i) = 0$, further ignoring the global influence $\eta(\mathbf{s}_i, \mathbf{s}_j) = 0$ and setting the local influence $\zeta(\sigma_{i,j}, \mathbf{s}_i, \mathbf{s}_j) = z_{i,j}\psi_{i,j}\mathbf{\Theta}_{i,j}$ the state kinematics generalised by equation (1) reduces to

$$\dot{\mathbf{s}}_i = \sum_{j \neq i} z_{i,j}\psi_{i,j}\mathbf{\Theta}_{i,j} \tag{3}$$

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The gain evolutions suggested by equation (2) are constructed by choosing a gain threshold switch τ and incorporating a growth factor α and a decay factor β should the corresponding states lie within or without the local group of n.

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$$\dot{\sigma}_{i,j} = g_{i,j} \tag{4}$$

where

$$g_{i,j} = \begin{cases} \alpha & \text{if } \|\Delta_{i,j}\| \in \Omega_i, \ \sigma_{i,j} < \tau \\ -\beta & \text{if } \|\Delta_{i,j}\| \notin \Omega_i, \ \sigma_{i,j} > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Networks

We shall couch our discussion in the language of *complex network theory*. A *network* is a weighted graph, that is, a set of elements called *nodes* or *vertices*, which may be connected to one another via relational links (*edges*). To each node we assign a *state* and to each edge a weight (or *gain*), $\sigma_{i,j}$.

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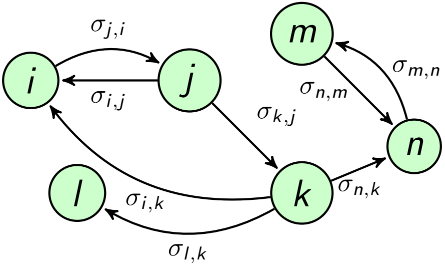

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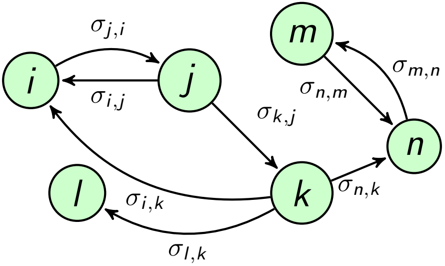

We want our states and gains to evolve until some "configuration"

Defining a flock

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For a configuration of n nodes, we consider a square lattice of $l = (2n-1)^2 - 1$ nodes centred around a target location. Next we measure the number of nodes within this lattice (or within a radius of $d\sqrt{2l}$).

Adjacency matrices

We shall manipulate the adjacency matrix to glean information about connected components in our spatial networks. Connected nodes represent agents in close proximity whereas disconnected nodes represent agents farther apart than our flocking target distance.

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Consider a finite symmetric graph G(V,E) with n vertices $v_i \in V$ and c edges $e_{i,j} \in E$. The adjacency matrix A describes the $2c$ arcs (or c edges). Where

$$A \ni a_{i,j}, \qquad a_{i,j} = \begin{cases} 0 & \text{if } i = j, \\ 1 & \text{if } e_{i,j} \in E \\ 0 & \text{otherwise.} \end{cases}$$

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Reflexive adjacency

Let $S \ni s_{i,j}$ be $A + 2I_n$, the reflexive adjacency containing *self loops* on each node, hence

$$s_{i,j} = \begin{cases} 2 & \text{if } i = j, \\ 1 & \text{if } e_{i,j} \in E \\ 0 & \text{otherwise.} \end{cases}$$

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Now S^c catalogues the number of distinct walks of length c from each node v_i to v_j, $\forall v_i, v_j \in V$. It follows that for a given row k in S^c, if column l is non-zero then $e_{k,l} \in E$, otherwise v_k and v_l belong to different components.

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