Acoustics of the Brain

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- The aim of my project is to accurately determine the mechanical properties of the brain using small-amplitude elastic waves.
- Because the brain is very soft, conventional methods like tensile tests are destructive and unreliable. These new methods are non-destructive and can be applied in vivo.
- The work will rely on the theory of acousto-elasticity, which essentially measures the stiffness of the tissue by linking it to the speed of the wave travelling in it.
- This may have important consequences for neurosurgery and for the simulation of traumatic brain injury.

Non-linear elasticity

Consider a solid initially at rest in the reference configuration $\mathcal{B}_0$. It is then brought to an equilibrium configuration (the current configuration $\mathcal{B}$, say).

The deformation $\mathbf{x} = \chi(\mathbf{X})$ is described by the **deformation gradient F**:

$$\mathbf{F} = \frac{\partial \boldsymbol{\chi}}{\partial \mathbf{X}}, \qquad F_{i\alpha} = \frac{\partial \chi_i}{\partial X_\alpha}. \tag{1}$$

For **incompressible hyperelastic solids**,

$$\mathbf{S} = \frac{\partial W}{\partial \mathbf{F}} - p\mathbf{F}^{-1}, \tag{2}$$

where $\mathbf{S}$ is the nominal stress tensor, W is the strain energy density function and p is a Lagrange multiplier associated with the constraint of incompressibility, $\det \mathbf{F} = 1$.

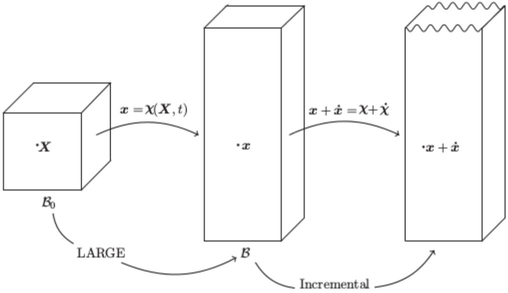
The **equations of equilibrium**, in the absence of body forces, are

$$\mathrm{Div}\mathbf{S} = \mathbf{0}, \qquad \frac{\partial S_{\alpha i}}{\partial X_\alpha} = 0, \tag{3}$$

Incremental elasticity

To model wave propagation we perturb the position by a small "incremental" displacement

$$\dot{\mathbf{x}} = \dot{\chi}\left(\chi\left(\mathbf{X}\right), t\right) \equiv \mathbf{u}(\mathbf{x}, t). \tag{4}$$



Then, linearising the equations of motion, we obtain

$$\frac{\partial \Sigma_{ji}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{5}$$

where $\Sigma_{ji} = \mathcal{A}_{0jikl} u_{l,k} + \dot{p}\delta_{ji} + p u_{j,i}$ and $\mathcal{A}_{0jikl} = \frac{\partial^2 W}{\partial F_{i\alpha} \partial F_{l\beta}} F_{j\alpha} F_{k\beta}$ are the instantaneous elastic moduli. For a homogeneous deformation (e.g. compression, extension, simple shear) **F** is constant so the moduli are constant.

Also, incremental incompressibility is given by

$$u_{i,i} = 0 \tag{6}$$

Wave motion

We then seek a plane wave solution of the form

$$\mathbf{u} = \mathbf{a} f(\mathbf{n} \cdot \mathbf{x} - vt), \; \dot{p} = q f(\mathbf{n} \cdot \mathbf{x} - vt), \tag{7}$$

where $\mathbf{a}$ is the polarization vector and $\mathbf{n}$ is the direction of propagation.

Substituting into the incremental equations of motion, we eventually obtain the eigenvalue problem

$$\mathbf{Q}_0^*(\mathbf{n})\mathbf{a} = \rho v^2 \mathbf{a} \tag{8}$$

where $\mathbf{Q}_0^*(\mathbf{n}) = \mathbf{Q}_0(\mathbf{n}) - \mathbf{n} \otimes \mathbf{Q}_0(\mathbf{n})$ and $[\mathbf{Q}_0(\mathbf{n})]_{ij} = \mathcal{A}_{0piqj} n_p n_q$.

We find that the problem has a zero eigenvalue corresponding to eigenvector $\mathbf{n}$, so we disregard this solution. Therefore two solutions may exist, and they must be shear waves ($\mathbf{a} \cdot \mathbf{n} = 0$) in order to satisfy incremental incompressibility.

Acoustoelasticity

Now we take the underlying deformation to be uniaxial extension or compression in the x_1 direction with stretch $\lambda = 1 + e$.

Then we seek a plane harmonic wave solution, $f = e^{ik(\mathbf{n}\cdot\mathbf{x} - vt)}$ where $\mathbf{n} = [0, cos(\theta), -sin(\theta)]^T$, to obtain

$$\rho v^2 = \gamma_{21} cos^2(\theta) + \gamma_{31} sin^2(\theta), \tag{9}$$

where $\gamma_{21} = \mathcal{A}_{02121}$ and $\gamma_{31} = \mathcal{A}_{03131}$.

We also consider the strain energy density of fourth-order elasticity

$$W = \mu tr(\mathbf{E}^2) + A/3 tr(\mathbf{E}^3) + D(tr(\mathbf{E}^2))^2, \tag{10}$$

where $\mathbf{E}$ is the Green strain tensor.

Acoustoelasticity

For this W, we may then expand $\gamma_{21} = \gamma_{31}$ in powers of e to obtain

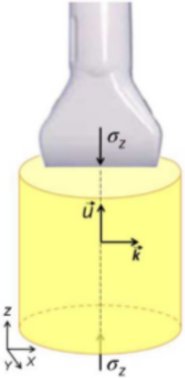
$$\rho v^2 = \mu + \frac{A}{4}e + (2\mu + A + 3D)e^2, \tag{11}$$

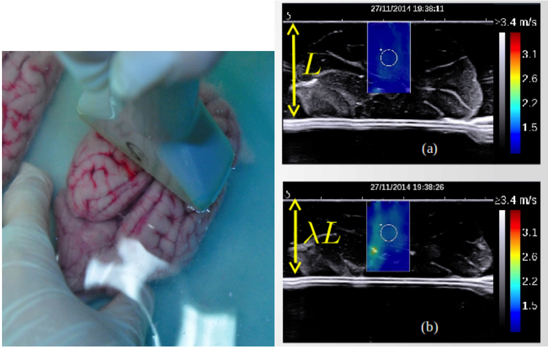
Hence we can determine the material parameters by deforming the material while measuring the wave speed.

Supersonic Shear Imaging

Supersonic Imagine's Aixplorer device uses acoustic radiation force to create a shear wave. It also generates a real-time ultrasound image. The wave can be seen in the image and its speed measured.

When the tissue is compressed, the speed changes, and we can use acousto-elasticity theory to determine the material parameters.

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