Earthquakes as Stick-Slip systems: application of Filippov Theory

An introduction

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Filippov systems outline

A Filippov system is a non-linear system whose dynamics is described by a discontious vector field.

$$\dot{\underline{x}} = \begin{cases} f_1(\underline{x}) & if\ \sigma(\underline{x}) > 0 \\ f_2(\underline{x}) & if\ \sigma(\underline{x}) < 0 \end{cases}$$

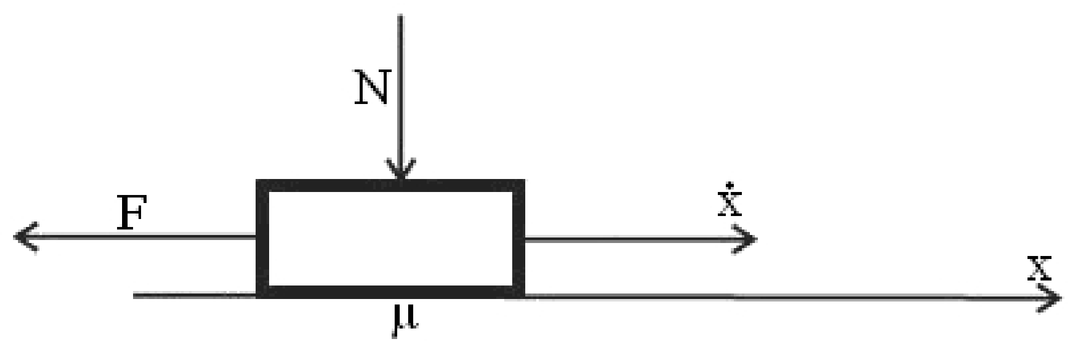
Where $\sigma(\underline{x})$ is called *switching manifold* and it's the implicit equation of the boundary manifold between the regions where the vector fields are continuous and differentiable.

Stick-Slip phenomenon - overview

Stick-slip is a spontaneous jerking motion that can occur while two objects are sliding over each other.

The main actor of stick-slip is the *Coulomb dry friction*:

$$F = \mu N$$



Where N is the closing force:

$$N = m g$$

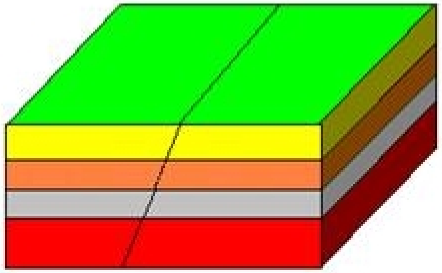
and μ is the kinetic friction coefficient.

Stick-Slip phenomenon - examples

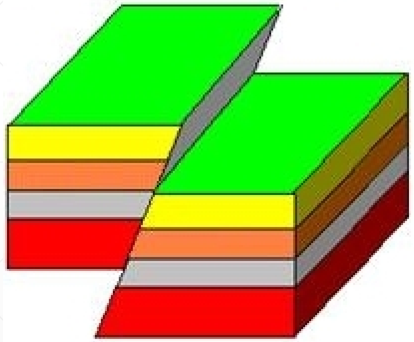
- Screetch of chalks on blackboard
- Creak of a door
- Noise of car brakes
- Belt squeal of a car
- Noise of a train stopping
- Sliding of the bow on a violin string

Stick-Slip phenomenon - earthquakes

Let's consider a seismogenic fault made by two coupled segments, subject to a constant strain rate.



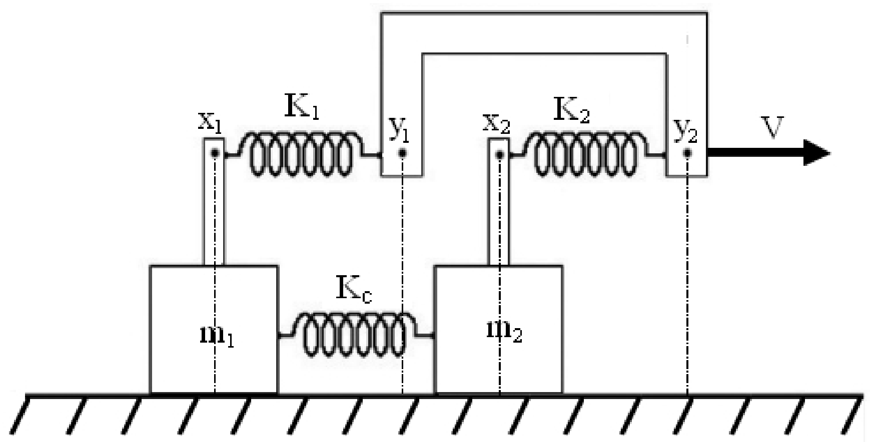
In static conditions the segments are sticked. The strain makes to increase the applied force that is counterbalanced by the dry friction between the contact surfaces.



When the applied force is too big an earthquake occurs. The segments slide over each other until they reach a new configuration.

So earthquakes can be considered as stick-slip systems.

Modeling – overall model



The system can be modelled as a 2-DOF mechanical system.
Two mass m_1 and m_2 coupled by a spring K_c are connected to the forcing respectively through springs K_1 and K_2.

Applying the law of the balance of the forces, we have:

$$m_1\ddot{x}_1 - F_{s_1} - F_{s_c} = F_{f_1}$$
$$m_2\ddot{x}_2 - F_{s_2} + F_{s_c} = F_{f_2}$$

With:

F_{s_i} $i = 1,2,c$ elastic restoring force
F_{f_i} $i = 1,2$ friction force

Modeling – overall model

Let $\begin{cases} z_1 = x_1 \\ z_2 = \dot{x}_1 \end{cases}$ $\begin{cases} w_1 = x_2 \\ w_2 = \dot{x}_2 \end{cases}$ and let L_1, L_2 and L_c the rest length of the springs,

then the vector field is defined by the following function $f: \mathbb{R}^6 \to \mathbb{R}^6$

$$f = \begin{cases} y_2 = v \\ \dot{z}_1 = z_2 \\ \dot{z}_2 = \dfrac{1}{m_1}\left[K_1(y_1 - z_1 - L_1) + K_c(w_1 - z_1 - L_c) - F_{f_1} sgn\,(z_2)\right] \\ \dot{w}_1 = w_2 \\ \dot{w}_2 = \dfrac{1}{m_2}\left[K_2(y_2 - w_1 - L_2) - K_c(w_1 - z_1 - L_c) - F_{f_2} sgn\,(w_2)\right] \end{cases}$$

Modeling – switching manifolds

Dry friction is:

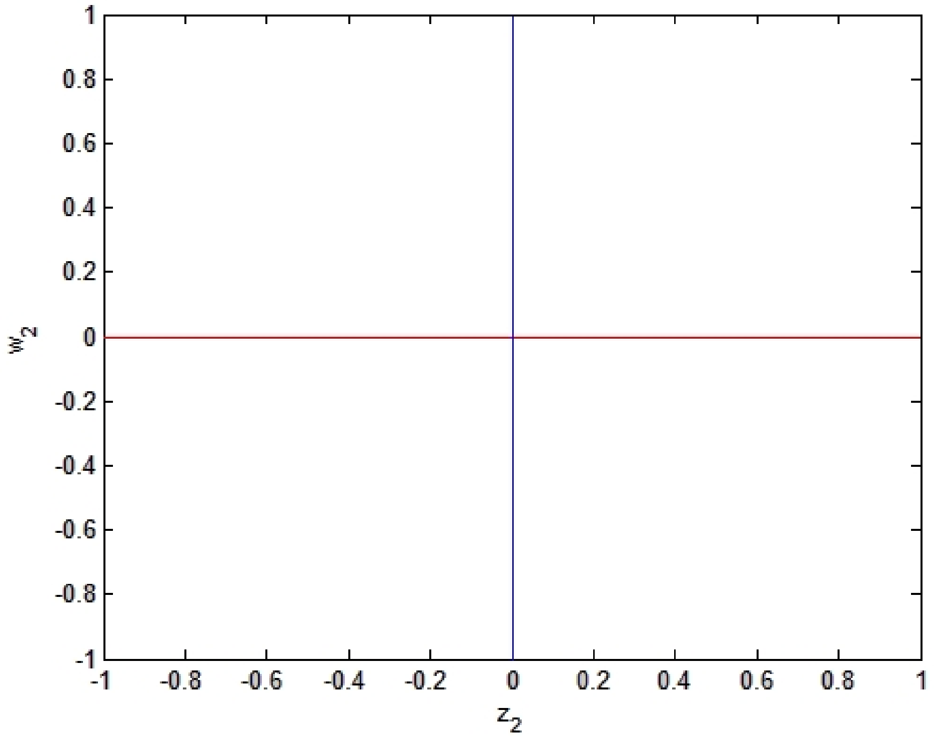
$$F_1 = \begin{cases} -F_{f_1} \; if \; z_2 > 0 \\ +F_{f_1} \; if \; z_2 < 0 \end{cases}$$

$$F_2 = \begin{cases} -F_{f_2} \; if \; w_2 > 0 \\ +F_{f_2} \; if \; w_2 < 0 \end{cases}$$

Than there are two switching manifold:

$$\sigma_1 = z_2 \qquad \sigma_2 = w_2$$

Projecting these manifolds in the $z_2 w_2$subspace, we clearly obtain the coordinate axes



Modeling – equilibrium conditions

In the subspace $z_1 w_1$ applying the following static conditions:

$$z_1 = const \rightarrow \dot{z}_1 = \ddot{z}_1 = 0$$
$$w_1 = const \rightarrow \dot{w}_1 = \ddot{w}_1 = 0$$

We find the following two lines:

$$\begin{cases} w_1 = \left(1 + \dfrac{K_1}{K_c}\right) z_1 + \dfrac{K_1}{K_c}(L_1 - y_1) + L_c \\ w_1 = \left(\dfrac{K_c}{K_2 + K_c}\right) z_1 + \dfrac{K_2}{K_2 + K_c}(y_2 - L_2) + \dfrac{K_c}{K_2 + K_c} L_c \end{cases}$$

That involve an equilibrium conditions between the srpings acting on each mass:

$$\begin{cases} \dfrac{K_1}{K_c} = \dfrac{w_1 - z_1 - L_c}{z_1 - y_1 + L_1} \\ \dfrac{K_2}{K_c} = \dfrac{z_1 - w_1 + L_c}{w_1 - y_2 + L_2} \end{cases}$$

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