$$\left.\begin{aligned} \boxed{2}x + 3y + 2z &= 100 \\ x + y + 4z &= 70 \\ 20x + 10y + 10z &= 500 \end{aligned}\right\}$$ system of linear equations

The system is equivalent to the following system:

$$\left[\begin{aligned} R_2 &\to R_2 - \tfrac{1}{2}R_1 \\ R_3 &\to R_3 - 10R_1 \end{aligned}\right]$$

$$\left.\begin{aligned} 2x + 3y + 2z &= 100 \\ \boxed{-\tfrac{1}{2}}y + 3z &= 20 \\ -20y - 10z &= -500 \end{aligned}\right\}$$ second system

The second system of equations is equivalent to:

$$\left[R_3 \longmapsto R_3 - 40R_2\right]$$

$$\begin{aligned} 2x + 3y + 2z &= 100 \\ -\tfrac{1}{2}y + 3z &= 20 \\ -130z &= 130 \end{aligned}$$

Back substitution:

$z = 10$

$y = 20$

$x = 10$

Notation

2 is the pivot in the first stage

$-\frac{1}{2}$ is the pivot in the second stage

The above procedure for solving a system of linear equations is called gaussian elimination.

Q) Does this procedure always work?

A) No. It doesn't work if a pivot at some stage is zero.

Topic 3

Determinants, eigenvalues & eigenvectors

(Mainly 2×2 matrices)

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Definition The adjoint matrix of A is

$$\operatorname{adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Observe

$$A \cdot \operatorname{adj}(A) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix} = (ad-bc)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Definition The determinant of the 2x2 matrix A is the number

$$\det(A) = ad - bc.$$

Note:

$$A \cdot \operatorname{adj}(A) = \det(A) \cdot I$$

or $$A\left(\frac{1}{\det(A)} \operatorname{adj}(A)\right) = I$$

Thus:

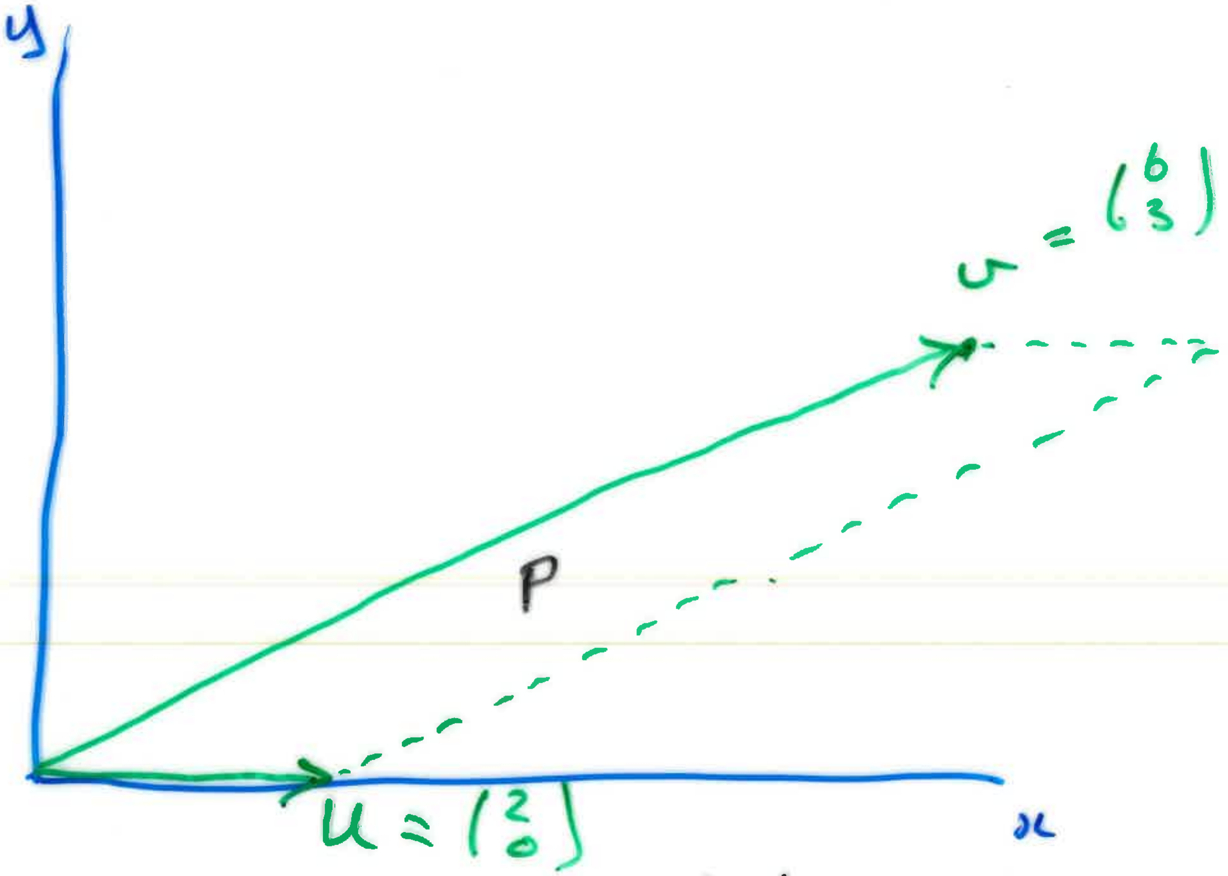
Proposition If $\det(A) \neq 0$ then

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

for any 2x2 matrix A.

Consider two "random" vectors

$$u = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \quad v = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$



So u and v determine a parallelogram P.

Area of P = base × ⊥ height

$= 2 \times 3$

$= 6$

Consider

$$A = \begin{pmatrix} 2 & 6 \\ 0 & 3 \end{pmatrix}$$

The vectors u, v can be thought of as the columns of a matrix.

$$\det(A) = 2.3 - 0.6 = 6.$$

$$\boxed{\begin{array}{l}\underline{\text{Theorem}}\ \text{The determinant of a } 2\times 2 \text{ matrix is equal to the } \pm \text{ area of} \\ \text{the parallelogram determined by its two columns.}\end{array}}$$