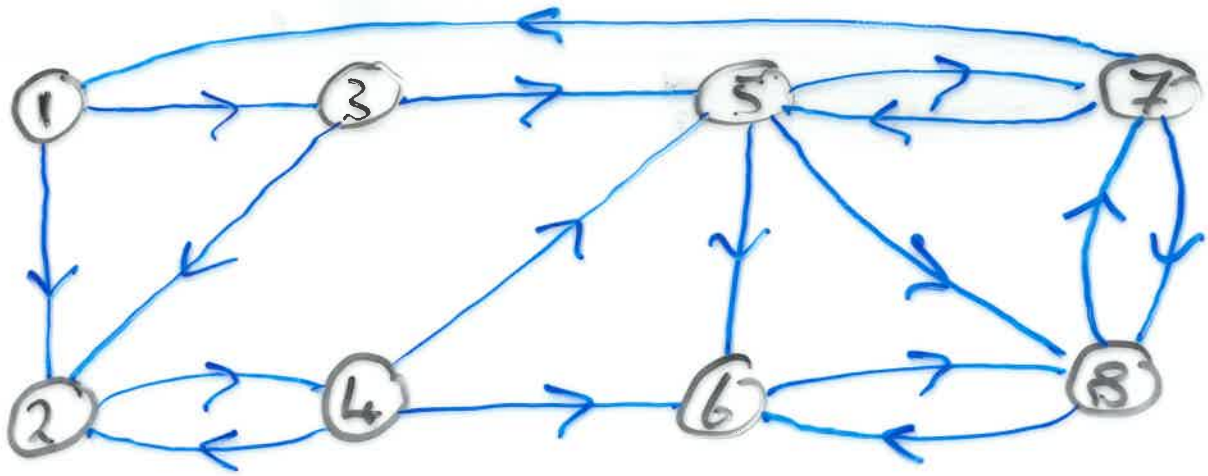
Google

A list of key words

eigenvalues, rabitts, google algorithm

results in a few web pages being listed as most likely of interest.

The WWW pages containing the words can be represented as a diagram of nodes (one node for each WWW page) and arrows (corresponding to a link from one page to another).



When listing pages Google first assigns a number I_n to each page P_n.

I_n is the "importance" of page P_n. Google lists the most important page first.

$$I_1 = \frac{I_7}{3}$$

$$I_2 = \frac{I_1}{2} + \frac{I_3}{2} + \frac{I_4}{3}$$

$$I_3 = \frac{I_1}{2}$$

$$I_4 = I_2$$

$$I_5 = \frac{I_3}{2} + \frac{I_4}{3} + \frac{I_7}{3}$$

$$I_6 = \frac{I_4}{3} + \frac{I_5}{3} + \frac{I_8}{2}$$

$$I_7 = \frac{I_5}{3} + \frac{I_8}{2}$$

$$I_8 = \frac{I_5}{3} + I_6 + \frac{I_7}{3}$$

How do we determine the numbers I_n.

$$
\underbrace{\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\
0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & \frac{1}{3} & 1 & \frac{1}{3} & 0
\end{pmatrix}}_{A}
\underbrace{\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{pmatrix}}_{v}
=
\underbrace{\begin{pmatrix} I_1 \\ I_2 \\ I \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{pmatrix}}_{v}
$$

Note: v is an eigenvector of A with corresponding eigenvalue $\lambda = 1$.

An eigenvector for A is:

$$
v = \begin{pmatrix} 0.0600 \\ 0.0675 \\ 0.0300 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{pmatrix}
\begin{array}{l} \\ * \\ \\ * \\ \\ \\ \\ \\ \end{array}
$$

Google list pages in the following order:

P_8

P_6

P_7

P_5

P_2

P_4

P_1

P_3

But: how do we calculate the eigenvectors for a square matrix A.

Let A be a 2×2 matrix.

Defn The polynomial

$$p_A(\lambda) = \det(A - \lambda I)$$

is called the characteristic polynomial of A.

Example $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$$p_A(\lambda) = \det\left(\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$= \det \begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix}$$

$$= (2-\lambda)(2-\lambda) - 1 \cdot 1$$

$$= \lambda^2 - 4\lambda + 4 - 1$$

$$= \lambda^2 - 4\lambda + 3.$$