2018 Q5 b)

i) A critical point occurs at those x where $f'(x) = 0$ or $f'(x)$ does not exist.

$$f'(x) = 8x^3 - 12x^2$$
$$= 4x^2(2x - 3)$$

$f'(x) = 0$ where $x = 0$ and $x = \frac{3}{2}$.

These are the two critical points.

ii) f increases when $f'(x) > 0$.

So f increases on

$$[\tfrac{3}{2}, \infty) = \{x \in \mathbb{R} : x \geq \tfrac{3}{2}\}$$

f decreases when $f'(x) \leq 0$.

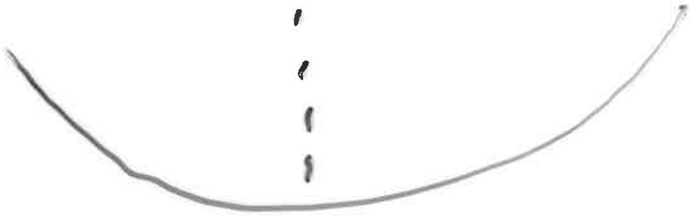
So f decreases on

$$(-\infty, \tfrac{3}{2}] = \{x \in \mathbb{R} : x \leq \tfrac{3}{2}\}.$$

Rough work

Max

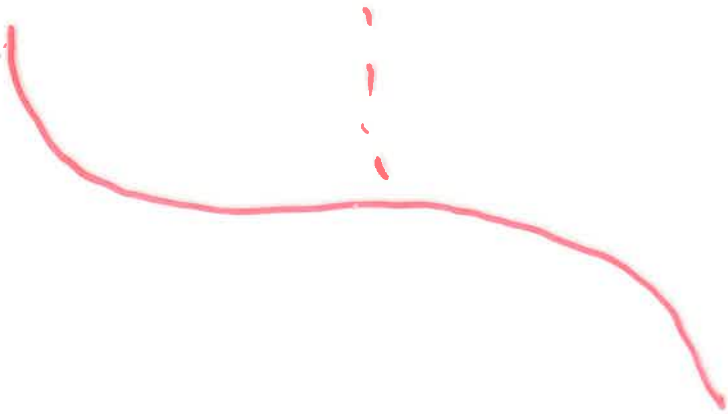
f' + + + + + - - - - -



Min

f' - - - - - + + + + + +

Please don't use a red pen on my exam!



neither

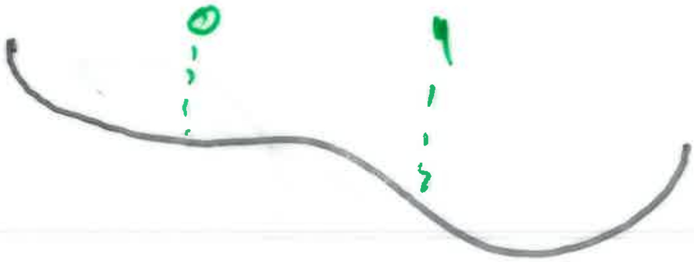
f' - - - - - 0 - - - -

5 b iii)

$x = 0$ is neither

$x = \frac{3}{2}$ is local min

5 b v) ~~f(x) is concave down between $x = 0$ and $x = \frac{3}{2}$.~~



$$f''(x) = 24x^2 - 24x$$
$$= 24x(x-1)$$

f(x) is concave down on

$[0, 1]$.

$$\frac{d}{dt} f(\text{something})$$

$$= f'(\text{something}) \cdot \frac{d}{dt}\text{something}$$

$$\frac{d}{dt} \ln(\text{something}) = \frac{1}{\text{something}} \cdot \frac{d}{dt}\text{something}$$

$$\frac{d}{dt} \ln(2t) = \frac{1}{2t} 2 = \frac{1}{t}$$

Rough work

6 a)

$$\int \frac{1}{\sqrt{t}}\,dt = \int t^{-\frac{1}{2}}\,dt$$

$$= 2t^{\frac{1}{2}} + c$$

$$\int t\cos(t^2)\,dt = \frac{1}{2}\sin(t^2) + c$$

$$\int \frac{t}{t^2+1}\,dt = \frac{1}{2}\ln(t^2+1)$$

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6 b) $y(t)$ = temp. at time t.

i) $$\frac{dy}{dt} = k(y-20).$$

ii) $$z = y - 20$$

$$\frac{dz}{dt} = \frac{dy}{dt} \quad \boxed{\frac{dz}{dt} = kz} \quad *$$

The solution to (*) has the form

$$z = A e^{kt}.$$

$t=0$: $60 = A e^{0} = A$

$$z = 60 e^{kt}$$

$t=5$ $30 = 60 e^{5k}$

$$\frac{1}{2} = e^{5k}$$

When is $y(t) = 40$ or $z(t) = 20$?

Need

$$20 = 60 e^{kt}$$

$$\frac{1}{3} = \left(e^{5k}\right)^{\frac{t}{5}}$$

$$\frac{1}{3} = \left(\frac{1}{2}\right)^{\frac{t}{5}}$$

$$\ln\left(\frac{1}{3}\right) = \frac{t}{5} \ln\left(\frac{1}{2}\right)$$

$$t = \frac{5 \ln\left(\frac{1}{3}\right)}{\ln\left(\frac{1}{2}\right)} .$$