2017-18

Q 2a)

we decipher using

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto A^{-1}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} E \\ S \end{pmatrix} = \begin{pmatrix} 4 \\ 18 \end{pmatrix} \longmapsto \begin{pmatrix} 9 & 5 \\ 3 & 14 \end{pmatrix}\begin{pmatrix} 4 \\ 18 \end{pmatrix} = \begin{pmatrix} 22 \\ 4 \end{pmatrix} = \begin{pmatrix} W \\ E \end{pmatrix}$$

mod 26

$$\begin{pmatrix} D \\ C \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \longmapsto \begin{pmatrix} 9 & 5 \\ 3 & 14 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 11 \\ 11 \end{pmatrix} = \begin{pmatrix} L \\ L \end{pmatrix}$$

first four letters of plaintext:

WELL

Intermediate Value Theorem

2017 Q 5a

Let $f : [a, b] \to \mathbb{R}$ be a continuous function. Suppose that $f(a) f(b) < 0$. Then there exists some $c \in [a, b]$ such that $f(c) = 0$.

$$f(x) = x^5 - 4x - 1$$

x	-2	-1	0		2	3
$f(x)$	-1	$+$	$-$		-1	$+$

So $f(x) = 0$ for some $x \in [-2, -1]$, and for some $x \in [-1, 0]$, and for some $x \in [2, 3]$. So $f(x) = 0$ has at least 3 solutions. A polynomial of degree 3 has at most three roots.

2016 Q3 b)

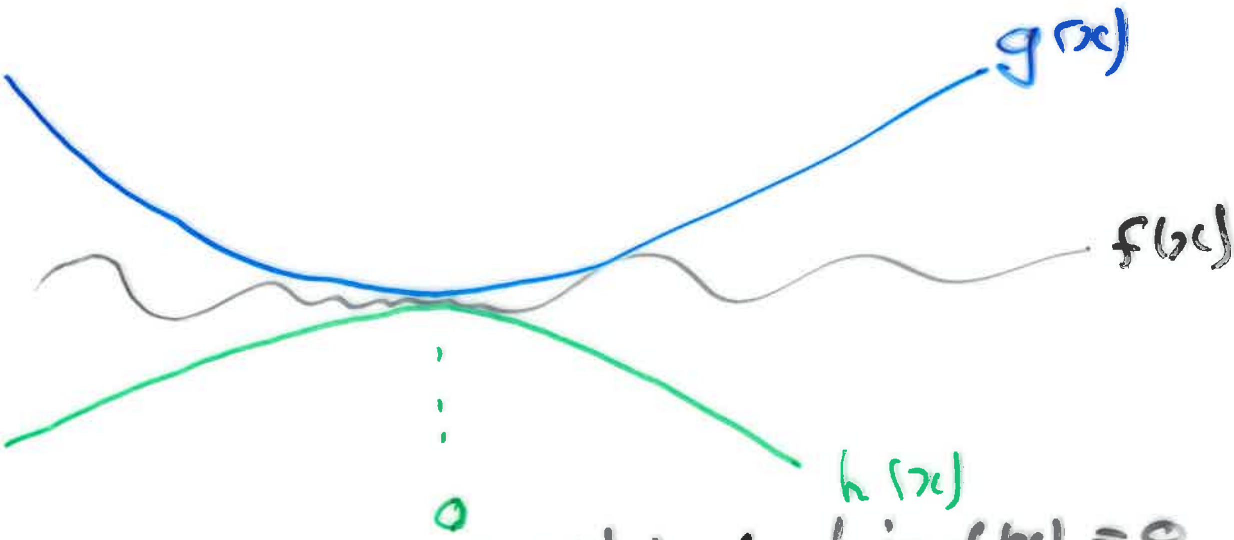
$$A\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}4 & 1\\2 & 3\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}9\\7\end{pmatrix}$$

Oops! it's not an eigenvector.

$$(2,1)\begin{pmatrix}4 & 1\\2 & 3\end{pmatrix} = (10, 5)$$

$\frac{1}{e^x}$

So $(2,1)$ is an eigenvector with eig. value $\lambda = 5$.



4b) $f(x) = x^2 e^{\sin(\frac{1}{x})}$ $\therefore \lim_{x\to 0} f(x) = 0.$

$g(x) = x^2 e$ $\quad \lim_{x\to 0} g(x) = 0$

$h(x) = x^2 \frac{1}{e}$ $\quad \lim_{x\to 0} h(x) = 0$