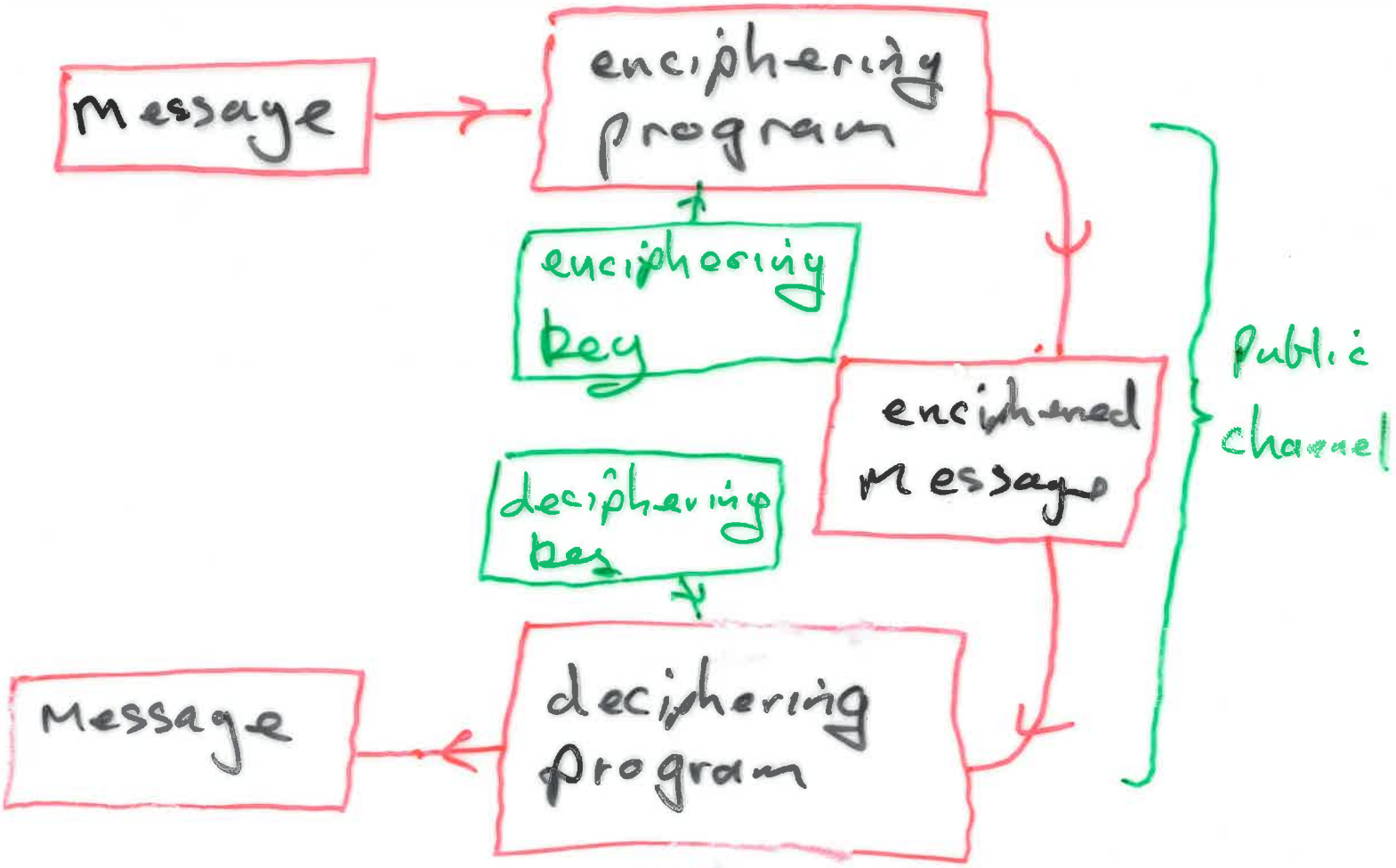
Cryptography



Basic Assumptions

1) Enciphering and deciphering programs are public knowledge.
2) keys are kept secret
3) Enciphered message will be intercepted.

Example

Receiver : PayPal

sender : you at home

channel : internet line & wifi

alphabet : A, B, C, ..., Z

plaintext : HELLO

Enciphering Program

A $\longleftrightarrow$ 1

B $\longleftrightarrow$ 2

C $\longleftrightarrow$ 3

⋮

Y $\longleftrightarrow$ 25

Z $\longleftrightarrow$ 0

alphabet = $\mathbb{Z}_{26}$ = numbers on a 26-hour clock

Enciphering key = E = $(3, 4)$

Enciphering Program:

$f_E : \mathbb{Z}_{26} \longrightarrow \mathbb{Z}_{26}, \; n \mapsto 3n + 4$

HELLO $\leftrightarrow$ 8 5 12 12 15

$\xrightarrow{f_E}$ 2 19 4 14 23

$\leftrightarrow$ B S N N W

Cipher text.

Deciphering key : Same pair of integers $D = (\alpha, \beta)$

Deciphering function :

$f_D : \mathbb{Z}_{26} \to \mathbb{Z}_{26}, \; n \mapsto \alpha n + \beta$

In this example with $E = (3, 4)$ what should $D = (\alpha, \beta)$ be ?

Encipher

$$n \rightsquigarrow 3n \rightsquigarrow \underbrace{3n + 4}_{m}$$

Decipher

$$m \rightsquigarrow m - 4 \rightsquigarrow 3^{-1}(m - 4)$$

What is $3^{-1} \bmod 26$

Answer $3^{-1} \equiv 9 \pmod{26}$

because $3 \cdot 9 \equiv 1 \pmod{26}$.

Deciphering function:

$$\begin{aligned} f_D(m) &= 3^{-1}(m - 4) \pmod{26} \\ &= 9(m - 4) \pmod{26} \\ &= 9m - 10 \\ &= 9m + 16 \end{aligned}$$

Deciphering key is $D = (9, 16)$.