Calculus

Topics

1) Functions & limits

2) Rates of change

3) Differential Equations

Text: Stewart, "calculus"

Quick introduction to (1) & (2)

La Tour Eiffel

↕ stone falls y metres in t seconds

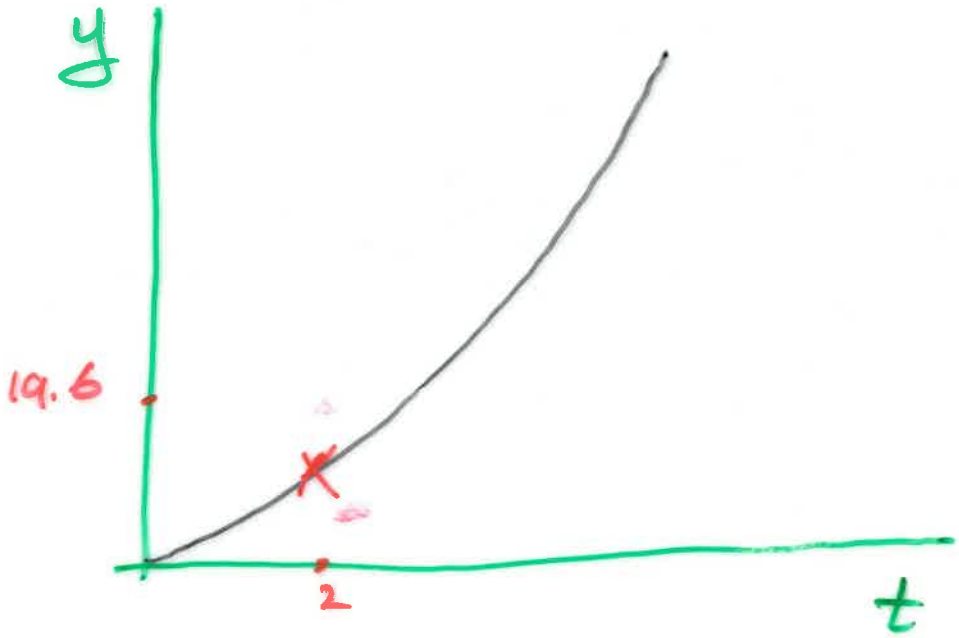
Experiment suggests

$$y = 4.9t^2$$

We say **y is a function of t.**

This means that for each value of t there is **one** corresponding value of y.

Functions are represented by their graph.



Question what is the average speed of the stone between $t=2$ and $t=3$ seconds?

Soln

$$\text{Average Speed} = \frac{\text{distance travelled}}{\text{time}}$$

$$= \frac{y(3) - y(2)}{1}$$

$$= \frac{4.9(9-4)}{1} = 4.9 \times 5$$

$= 24.5 \text{ m/s}$

Question What is the speed of the stone at time $t=2$?

The speed at $t=2$ is

$$v(2) = \lim_{h\to 0} \frac{y(2+h) - y(2)}{h}$$

$$= \lim_{h\to 0} \frac{4.9(2+h)^2 - 4.9(2^2)}{h}$$

$$= \lim_{h\to 0} 4.9\left(\frac{4+4h+h^2-4}{h}\right)$$

$$= \lim_{h\to 0} 4.9\left(\frac{4h+h^2}{h}\right)$$

$$= \lim_{h\to 0} 4.9(4+h)$$

$$= 19.6 \text{ m/sec}.$$

Consider

$$f(x) = \frac{\cos(3x\pi)}{2} + \frac{\cos(3^2 x\pi)}{2^2} + \frac{\cos(3^3 x\pi)}{2^3} + \cdots$$

$$f(x) = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{\cos(3^n x\pi)}{2^n}$$

Karl Weierstrass (1872) explained why $f(x)$ is everywhere continuous but differentiable nowhere !

Émile Picard : Newton would not have developed Calculus if he had known of such function.

The function $f(x)$ contradicted previous results of Ampère who used vague definitions.

Our aim: understand calculus rigorously