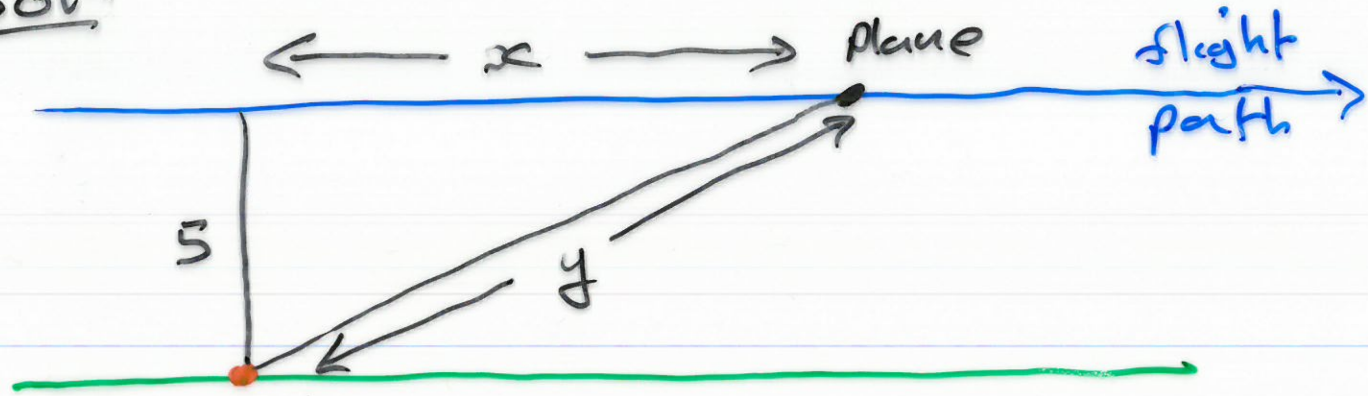
The the real stuff: APPLICATIONS

The derivation can be thought of as a rate of change.

Problem

An aircraft is flying horizontally at a speed of 600 km/h. How fast is the distance between the aircraft and a radio beacon increasing 1 minute after the plane passes 5 km above the beacon?

Sol^n



Want to find

$$\boxed{\frac{dy}{dt} \quad \text{when} \quad t = 1}$$

when $t = 1$, $x = 10$.

$$\boxed{5^2 + x^2 = y^2 \quad (*)}$$ for all t.

Chain Rule

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

Differentiate both sides of (*) with respect to t.

$$\boxed{2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt} \qquad **}$$

$$\frac{dx}{dt} = 10$$

when $t=1$ equation (**) becomes

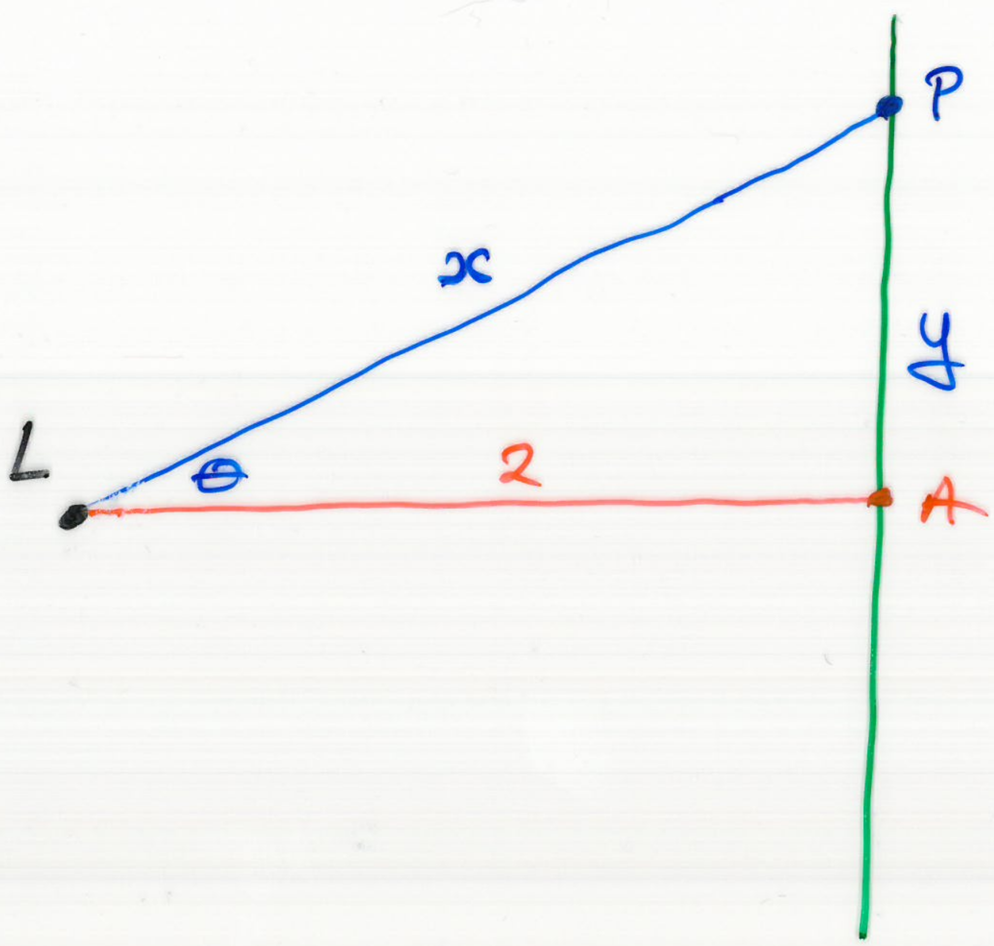
$$2 \cdot 10 \cdot 10 = 2\sqrt{5^2+10^2}\,\frac{dy}{dt}$$

$$\frac{200}{2\sqrt{125}} = \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{100}{\sqrt{5\cdot 25}} = \frac{100}{\sqrt{5}\sqrt{25}}$$

$$\frac{dy}{dt} = \frac{20}{\sqrt{5}} \text{ km/min}$$

Problem A lighthouse L is located on a small island 2 km from the nearest point A on a long straight shoreline. The lighthouse light rotates at 3 revs per minute. How fast is the illuminated spot P on the shoreline moving when it is 4 km from A?



Need to find

$$\boxed{\frac{dy}{dt} \quad \text{when} \quad y = 4.}$$

$\frac{d\theta}{dt} = 6\pi$ radians/min

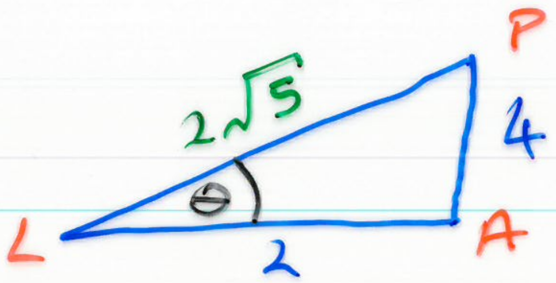
Both θ and y are functions of t.

$$\tan\theta = \frac{y}{2} \quad (*)$$

Differentiate both sides of $(*)$ with respect to t.

$$\boxed{\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{2}\frac{dy}{dt} \quad (**)}$$

when $y = 4$



$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{2\sqrt{5}}{2}$$

$$= \sqrt{5}.$$

from (**)

$$\frac{dy}{dt} = 2\sec^2\theta\,\frac{d\theta}{dt}$$

$$= 2\,.\,5\,.\,6\pi$$

$$= 60\pi \text{ km/min}.$$