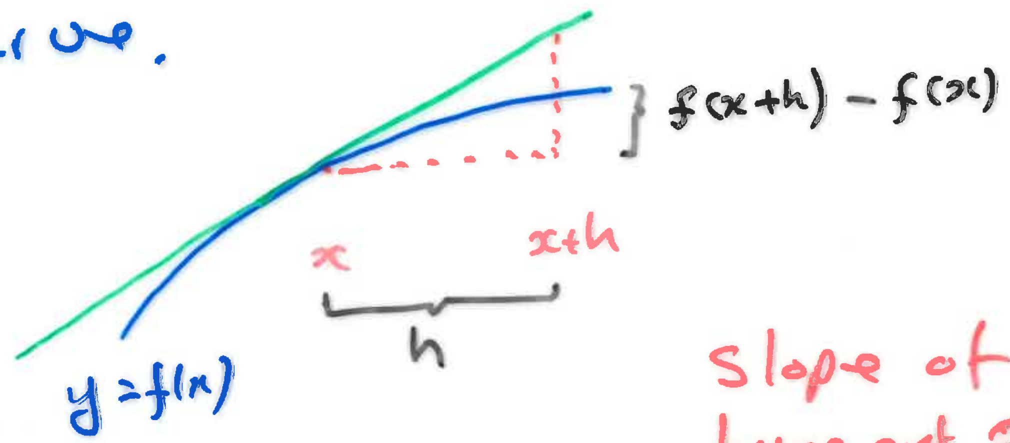
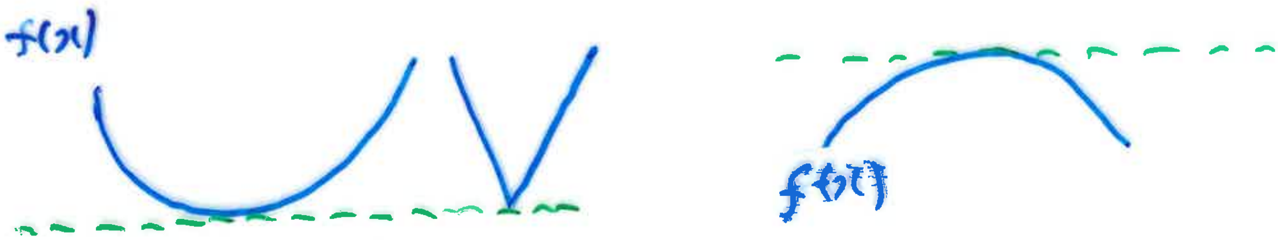
Applications II

Applications where derivatives are used to measure the slope of a tangent to a curve.



Slope of tangent $\simeq \dfrac{f(x+h) - f(x)}{h}$

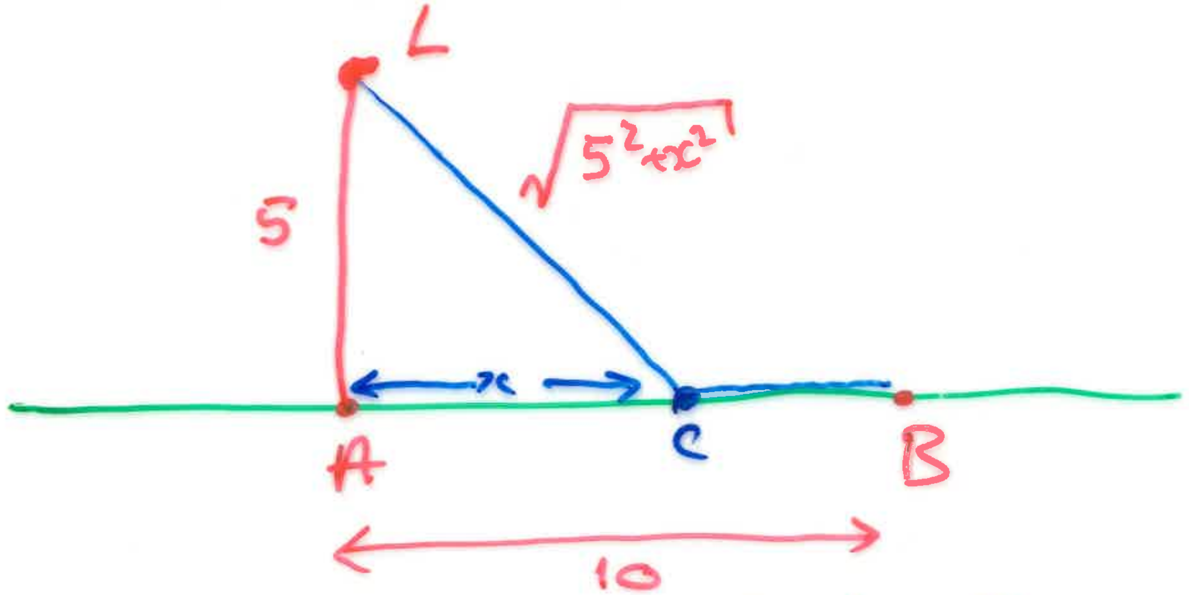
At points where a continuous function $f(x)$ is a local minimum or local maximum



we have that the derivative $f'(x) = 0$ or else $f'(x)$ does not exist.

<u>Problem</u> A lighthouse L is located on a small island 5 km North of a point A on a straight east-west coastline. A cable is to be laid from L to a point B on the coastline 10 km east of A. Laying the cable under water costs €5000 per kilometer. Laying it on land costs €3000 per kilometer.

<u>Question</u> What is the cheapest cost of laying the cable.



x = distance from A to C.

Let $f(x)$ = cost of laying the blue cable.

$$f(x) = 5000\sqrt{5^2+x^2} + 3000(10-x)$$

$$f(x) = 5000(5^2+x^2)^{\frac{1}{2}} + 3000(10-x)$$

$$f'(x) = \frac{5000}{\cancel{2}}(5^2+x^2)^{-\frac{1}{2}}\cancel{2}x - 3000$$

$$f'(x) = \frac{5000x}{\sqrt{5^2+x^2}} - 3000$$

$f'(x)$ exists for all $x \in \mathbb{R}$.

Now $f'(x) = 0$ when

$$3000 = \frac{5000\, x}{\sqrt{5^2 + x^2}}$$

$$3 = \frac{5x}{\sqrt{5^2+x^2}}$$

$$3\sqrt{5^2+x^2} = 5x$$

$$9(5^2+x^2) = 25x^2$$

$$9.5^2 = 16x^2$$

$$x^2 = \frac{3^2 . 5^2}{4^2}$$

$$x = \pm\frac{15}{4}$$

So $f'(x) = 0$ when $x = \frac{15}{4}$.

Common sense tells us that the minimum cost occurs when $x = \frac{15}{4}$.

The minimum cost of laying the cable is

$$\boxed{f\left(\frac{15}{4}\right) = 5000\sqrt{5^2+\frac{15^2}{4^2}} + 3000\left(10-\frac{15}{4}\right) \text{ euro.}}$$