Functions

A function $f: D \longrightarrow C$ consists of

1) a set D called the domain

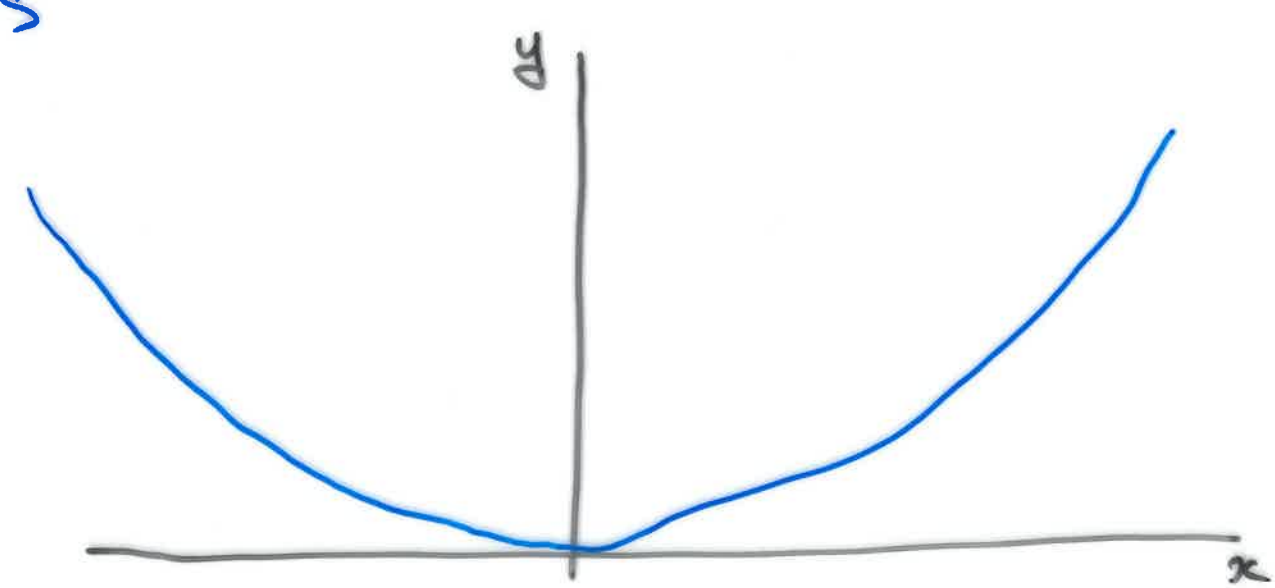
2) a set C called the codomain

3) a rule that assigns precisely one element $f(x) \in C$ to each input $x \in D$.

Example $f: \mathbb{R} \longrightarrow \mathbb{R}$ with $f(x) = x^2$

$D = \mathbb{R}$, $C = \mathbb{R}$

Us



The range of a function $f: C \to D$ is the set

$$\text{Range}(f) = \{ y \in C : y = f(x) \text{ with } x \in D \}.$$

Example (continued)

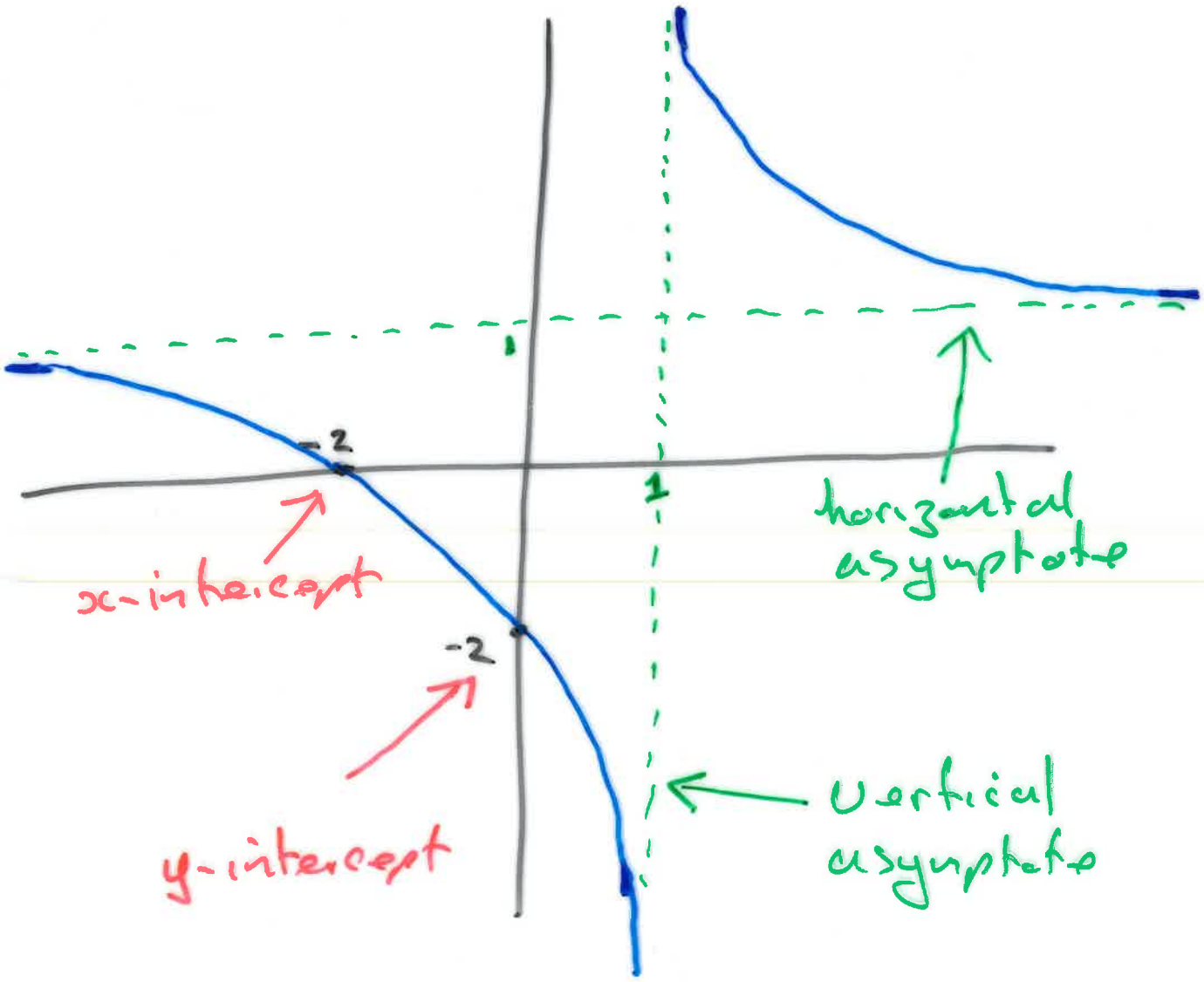
$f: \mathbb{R} \to \mathbb{R}, \; x \to x^2$

$$\text{Range}(f) = \{ x \in \mathbb{R} : x \geq 0 \} =: \mathbb{R}^{\geq 0}$$

Example $g : \mathbb{R} \setminus \{1\} \longrightarrow \mathbb{R}$

with $g(x) = \dfrac{x+2}{x-1}$.

$D = \mathbb{R} \setminus \{1\}$, $C = \mathbb{R}$



$\text{Range}(g) = \mathbb{R} \setminus \{1\}$

Convention

Often we use a formula such as

$$h(x) = \frac{x+2}{x-3}$$

to describe a function, without explicitly stating a domain or codomain.

In this situation we always assume that the domain is the largest possible subset of $\mathbb{R}$ for which the formula makes sense.

We take the codomain to be $\mathbb{R}$.

Examples

1) $h(x) = \dfrac{x+2}{x-3}$

$D = \mathbb{R} \setminus \{3\}$

$C = \mathbb{R}$

$\text{Range}(h) = \mathbb{R} \setminus \{1\}$

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2) $q(x) = \sqrt{x}$ positive square root of x

$D = \{x \in \mathbb{R} : x \geq 0\}$

$= [0, \infty)$

$C = \mathbb{R}$

$\text{Range}(q) = [0, \infty)$

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3) $p(x) = \sqrt{x^2 - 1}$

$C = \mathbb{R}$

$D = \{x \in \mathbb{R} : x \leq -1\} \cup \{x \in \mathbb{R} : x \geq 1\}$

$\text{Range}(p) = [0, \infty)$.

4) $r(x) = \sqrt{\frac{1}{x}}$

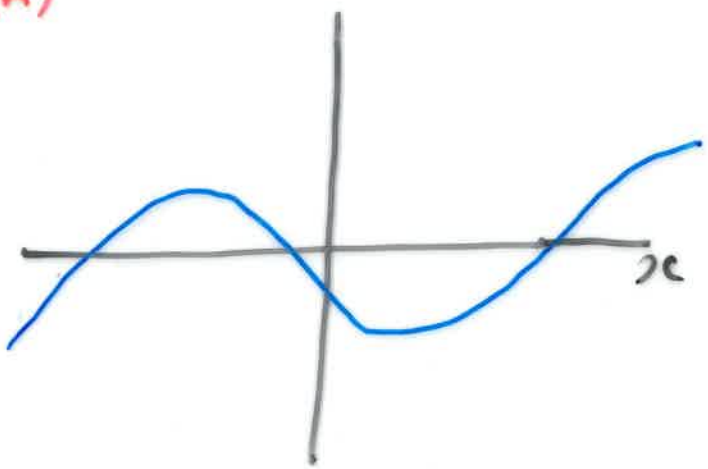
$D = \{x \in \mathbb{R} : x > 0\} =: (0, \infty)$

$C = \mathbb{R}$

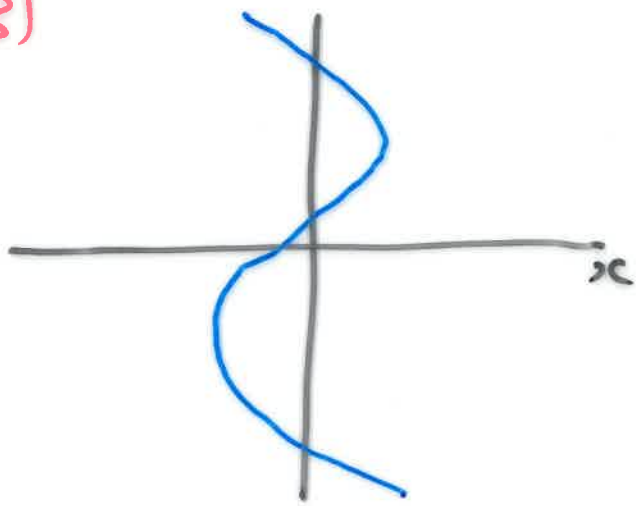
$\text{Range}(r) = \{x \in \mathbb{R} : x > 0\} = (0, \infty)$

Question Which of the following are graphs of functions of x?

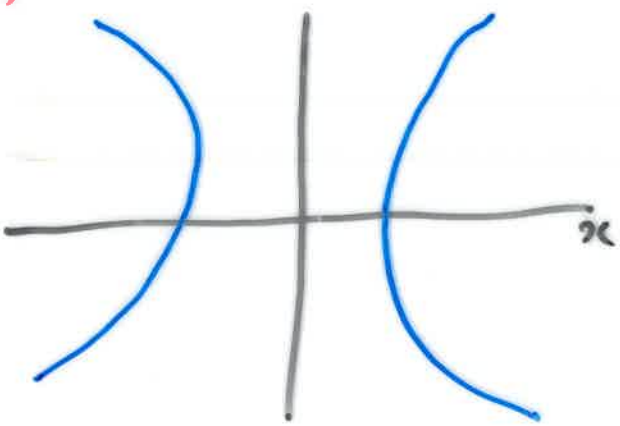
(A)



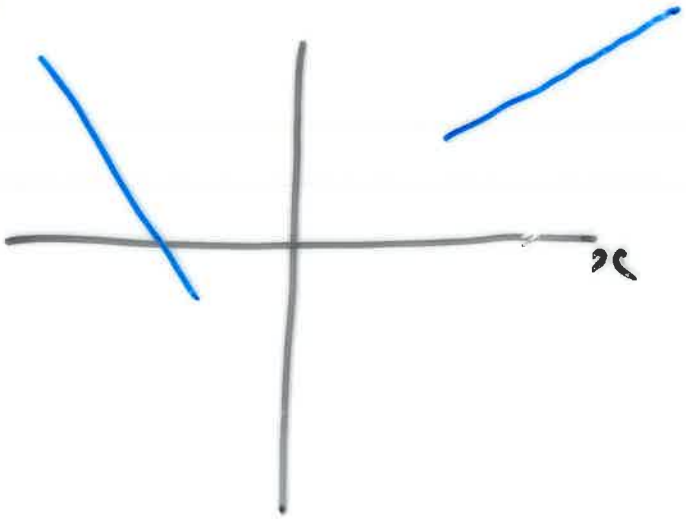
(B)



(C)



(D)



(B) is not a function

(C) is not a function

(A) is a function

(D) is a function, but its domain is a proper subset of $\mathbb{R}$.