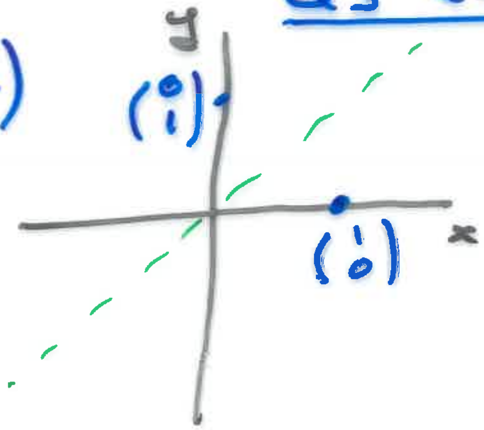
Please do not use a red pen in the exam!

Q3 2013

a)



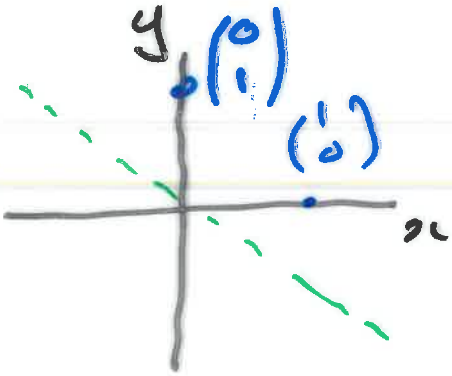
$$f\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

$$f\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\underbrace{\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}}_{F}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

F matrix of f



$$g\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$$

$$g\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\0\end{pmatrix}$$

$$\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix} = G \text{ matrix of } g$$

The matrix for $g \circ F$ is

$$GF = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Q3 b)

$$\begin{pmatrix} 1 & 4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix} = (-3)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

eigenvalue $\lambda_1 = -3$

$$\begin{pmatrix} 1 & 4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} = 2\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

eigenvalue $\lambda_2 = 2$.

Eigenvalues of A^{-1} are

$-\frac{1}{3}$ and $\frac{1}{2}$.

Q4a)(i)

$$\lim_{x \to 0} \frac{\sqrt{9+x} - 3}{x}$$

$$= \lim_{x \to 0} \frac{(\sqrt{9+x} - 3)}{x} \frac{(\sqrt{9+x} + 3)}{(\sqrt{9+x} + 3)}$$

$$= \lim_{x \to 0} \frac{\cancel{9} + x - \cancel{9}}{x(\sqrt{9+x} + 3)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{9+x} + 3} = \frac{1}{6}.$$

ii) $$\lim_{x \to 5^+} \frac{|x-5|}{x-5} = \frac{x-5}{x-5} = 1$$

$$\lim_{x \to 5^-} \frac{|x-5|}{x-5} = \frac{5-x}{x-5} = -1$$

Limit does not exist.

4 a) The function is always continuous for any $x \neq 2$.

At $x = 2$, need

$$x^3 + kx^2 + x = lx + k$$

for continuity. i.e.

$$10 + 4k = 2l + k$$

$$\boxed{10 + 3k = 2l} \quad \text{— } (1)$$

At $x = 2$, need

$$3x^2 + 2kx + 1 = l$$

for ~~cont~~ differentiability. i.e.

$$12 + 4k + 1 = l$$

$$\boxed{13 + 4k = l} \quad \text{— } (2)$$

$$\boxed{26 + 8k = 2l} \quad (2')$$

$(2') - (1)$: $\quad 16 + 5k = 0$

$$\boxed{k = \frac{-16}{5}}$$

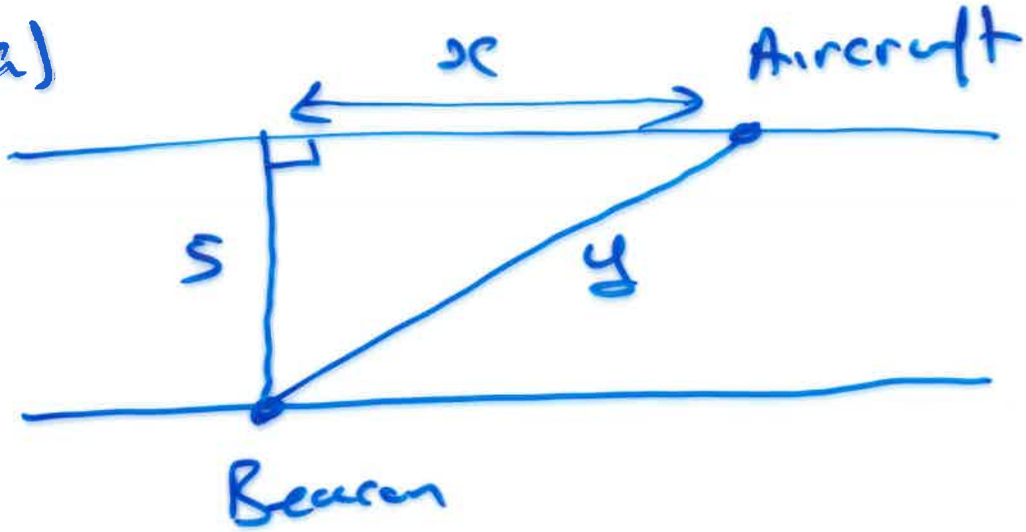
from (2)

$$13 - \frac{4.16}{5} = l$$

$$\frac{65 - 64}{5} = l$$

$$\boxed{l = \frac{1}{5}}$$

5 a)



Need to find $\frac{dy}{dt}$ at $t = 1$.

$$5^2 + x^2 = y^2$$

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} \quad (*)$$

$$2x = 2y\frac{dy}{dx}$$

$$\frac{x}{y} = \frac{dy}{dx}$$

Answer, $t = 1$,

$$(*) \quad \frac{dy}{dt} = \frac{10}{\sqrt{5^2+10^2}} \quad 10 \quad \text{km/minute.}$$