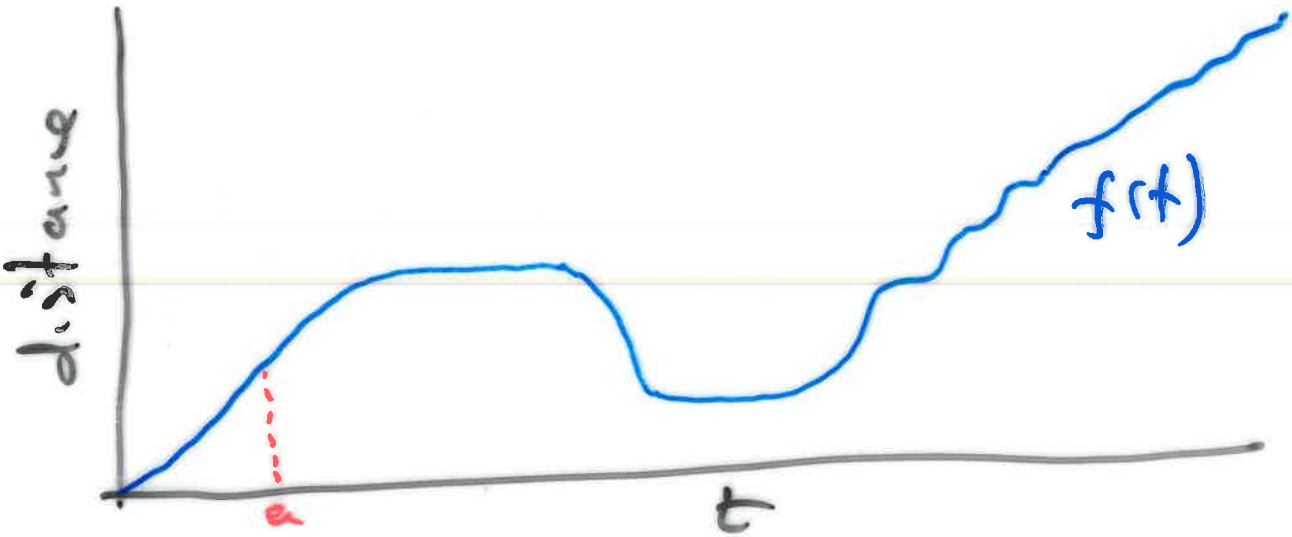
Second homework now online

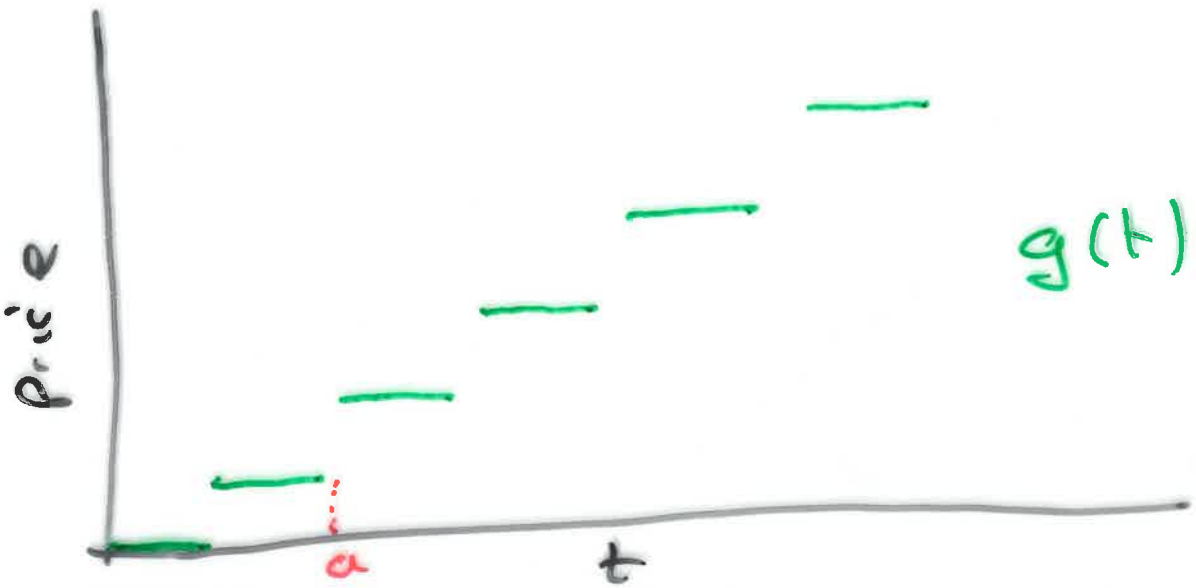
Continuity

h travel to Dublin Airport.

$f(t)$ = distance from Galway t minutes after leaving.

$g(t)$ = price of parking my car t minutes after entering the car park.





Intuitively: continuous means there are no breaks in the graph.

Better, more general, definition:

> A function $f(t)$ is continuous if a small change in the input yields only a small change in the output.

In the above example $f(t)$ is continuous, whereas $g(t)$ is not continuous.

In the context of functions

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$

the following is the best definition,

We say that $f(t)$ is continuous at a point $t=a$ if:

i) $f(a)$ is defined. (i.e. a must lie in the domain of f.)

ii) $\lim_{t \to a} f(t)$ exists, and

iii) $\lim_{t \to a} f(t) = f(a)$

Example Determine the constant k such that

$$f(x) = \begin{cases} x^3 & \text{for } x \geq 2 \\ kx & \text{for } x < 2 \end{cases}$$

is continuous at all points $x \in \mathbb{R}$.

Sol^n The only problem is at $x = 2$.

We need

$$\lim_{x \to 2} f(x) = f(2) = 8$$

i.e. We need

$$\lim_{x \to 2^-} f(x) = 8 = \lim_{x \to 2^+} f(x)$$

i.e. need

$$\lim_{x \to 2^-} kx = 8 = \lim_{x \to 2^+} x^3$$

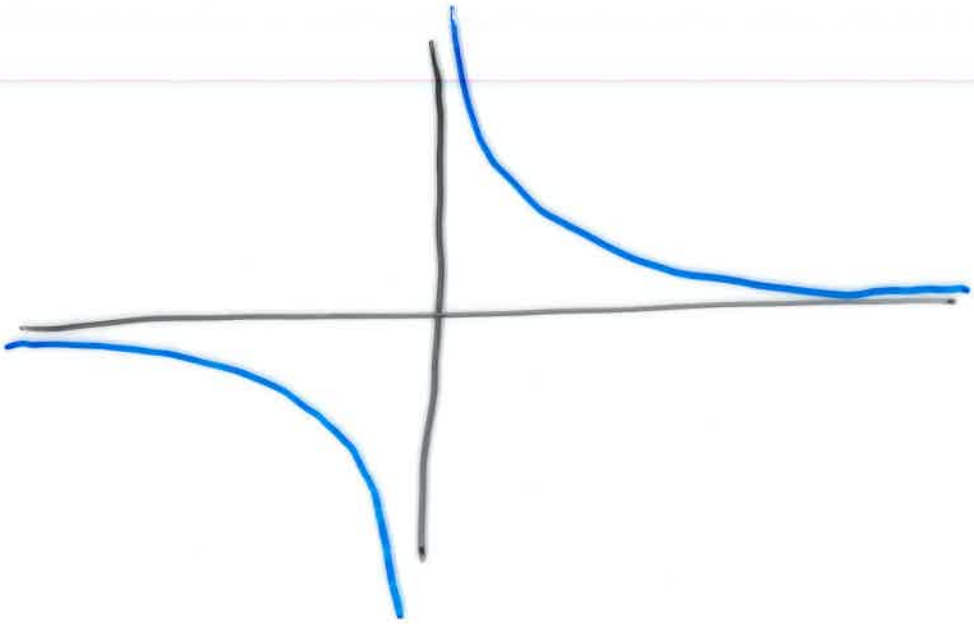
So we need

$$2k = 8$$

So we need $k = 4$.

Defn A function f is said to be **continuous** if it is continuous at all points x in its domain.

Example Is the function $f(x) = \frac{1}{x}$ continuous?



Yes!