Limits at infinity

Defn

$$\lim_{x \to \infty} f(x) = \lim_{x \to 0^+} f\left(\frac{1}{x}\right)$$

Defn

$$\lim_{x \to -\infty} f(x) = \lim_{x \to 0^-} f\left(\frac{1}{x}\right)$$

Example Evaluate

$$L = \lim_{x \to \infty} \frac{1}{\sqrt{x^2+2x} - x}$$

Sol^n

$$L = \lim_{x \to \infty} \frac{1}{\sqrt{x^2+2x} - x} \cdot \left(\frac{\sqrt{x^2+2x} + x}{\sqrt{x^2+2x} + x} \right)$$

$(A - B)$ $(A + B)$ $A^2 - B^2$

$$L = \lim_{x \to \infty} \frac{\sqrt{x^2+2x} + x}{x^2+2x - x^2}$$

$$L = \lim_{x \to \infty} \frac{\sqrt{x^2+2x} + x}{2x}$$

$$l = \lim_{x \to \infty} \frac{\sqrt{\frac{x^2 + 2x}{x^2}} + 1}{2}$$

$$\left[\text{Rough} \quad \frac{\sqrt{5^2}}{5} = \sqrt{\frac{5^2}{5^2}} = \sqrt{1} = 1 \right]$$

$$l = \lim_{x \to \infty} \frac{\sqrt{1 + \frac{2}{x}} + 1}{2}$$

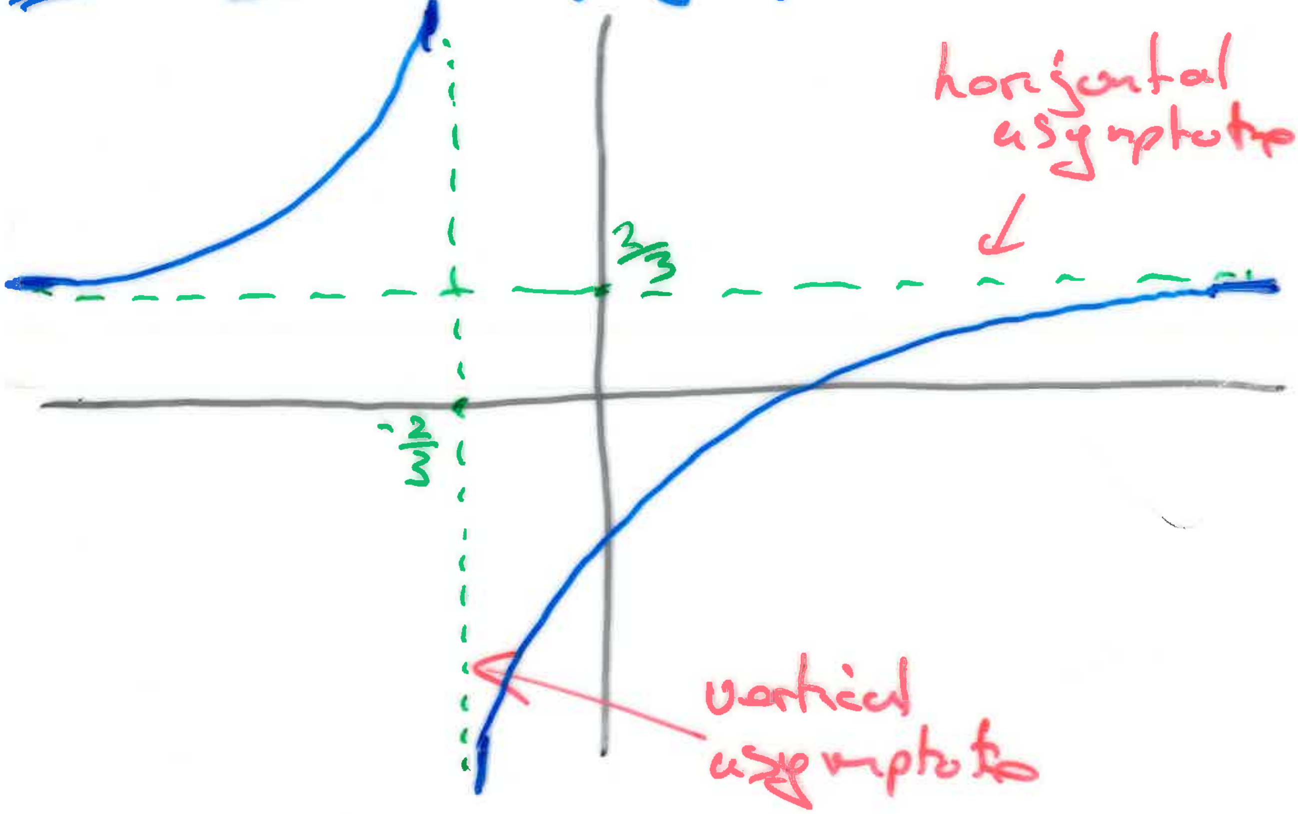
$$l = 1.$$

Example What are the horizontal and vertical asymptotes of

$$y = \frac{2x-5}{3x+2} \; ?$$

sketch the graph of y.

Sol^n Domain of $y = f(x)$ is $\mathbb{R} \setminus \{-\frac{2}{3}\}$.



$$\lim_{x \to \infty} \frac{2x-5}{3x+2} = \frac{2}{3}$$

$$\lim_{x \to -\infty} \frac{2x-5}{3x+2} = \frac{2}{3}$$

"So" limits at infinity correspond to horizontal asymptotes.

Note: Domain$(y=f(x)) = \mathbb{R}\setminus\{-\frac{2}{3}\}$

This corresponds to a vertical asymptote.

Note: $y=f(x)$ is a function

$f: \mathbb{R}\setminus\{-\frac{2}{3}\} \longrightarrow \mathbb{R}$

and it <u>is</u> continuous

Topic II

Rates of change & Differentiation

Given a function $f(x)$ we define the derivative to be a function $f'(x)$ defined as

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} .}$$

Example Find the derivative $f'(x)$ of the function $f(x) = x^2$.

Sol^n

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{(2x+h)h}{h}$$

$$= \lim_{h \to 0} 2x + h$$

$$= 2x.$$

So, for $f(x) = x^2$ we have

$f'(x) = 2x$.

Derivatives of Some basic functions

for $y = f(x)$ we'll often write

$$\frac{dy}{dx}$$

instead of

$$f'(x) .$$

- $\frac{d}{dx} x^n = n x^{n-1}$ for any $n \neq 0$.
- $\frac{d}{dx} \sin x = \cos(x)$
- $\frac{d}{dx} \cos(x) = - \sin(x)$
- $e = 2.71 \ldots$

  $$\frac{d}{dx} e^x = e^x$$
- etc.