RING THEORY (MA 416/MA 538) 2009-2010

Journal Task 1

First submission deadline for feedback : Tuesday September 29
There will be an opportunity for resubmission later following feedback.

BINARY OPERATIONS

Write a note of three to five pages in length on binary operations.

The tone and content of the note should be appropriate for a reader who has some knowledge of mathematics but who has not encountered the concept of a binary operation before. Your note should provide this hypothetical reader with a clear explanation of what a binary operation is, and should inform him/her of some important properties that a binary operation may or may not have. Plenty of examples should be provided.

In particular, your note should contain the following :

- a precise mathematical definition of a binary operation on a set, accompanied by some informal discussion.
- Some examples of binary operations, including answers to the following specific questions :
  - Why is subtraction not a binary operation on the set $\mathbb{N}$ of natural numbers?
  - Why is division not a binary operation on the set $\mathbb{Q}$ of rational numbers?
    Is division a binary operation on the set $\mathbb{Q}\backslash\{0\}$ of *non-zero* rational numbers?
- explanations of what it means for a binary operation to be
  - commutative
  - associative
- An explanation of what it means for a binary operation to have an *identity element*
- At least five different examples of binary operations on the set $\mathbb{Z}$ of integers, including
  - at least one commutative and at least one non-commutative example
  - at least one associative and at least one non-associative example
  - at least one example of an operation that has an identity element and at least one that does not.

  Your collection of examples should be accompanied by a discussion of whether each item is commutative, associative or contains an identity element.
- A few examples of binary operations on sets that are not sets of numbers - for example sets of matrices or sets of functions.
- Anything else that you consider appropriate.

NOTES

1. The layout and format of your journal entries are entirely up to you - in particular you don't have to address the points above in the order in which they are listed. Remember that each journal entry should be a coherent piece of writing and not just a disjointed list of points. Bear in mind that you are trying to communicate mathematical content to a prospective reader who may not already be familiar with the precise subject matter. Remember also, as in any piece of writing, to observe the basic rules of grammar, punctuation and sentence structure. Your journal entries do not have to be typed, but they should be legible and have high standards of presentation.

2. Thinking of examples of binary operations on the set of integers is not too hard. Addition, subtraction and multiplication are three obvious examples (division is not - why not?). You can also invent your own examples - for instance you could define a binary operation $\star$ on $\mathbb{Z}$ by declaring

$$x \star y = 2^{|x-y|}, \text{ for } x, y \in \mathbb{Z}.$$

   You can consider whether the operation $\star$ is commutative, associative or has an identity element in $\mathbb{Z}$. But don't use this example - make up your own.

3. You may find the following references helpful :

   (a) Sections 1.1 and 1.2 of *A First Course in Abstract Algebra*, by John A. Fraleigh (512.02 FRA).

   (b) Section 3, "Operations", in Chapter 1 of *Modern Algebra : an Introduction*, by John R. Durbin (512.02 DUR).

   (c) For definitions and examples, see

   `http://en.wikipedia.org/wiki/Binary_operation`;

   from there you can find links to wikipedia pages on commutativity, associativity, identity elements etc.
   If you are using wikipedia in your academic work, remember that while it is an extremely useful source of information it is not always an entirely reliable reference.

4. You can probably find textbooks or webpages with sections on binary operations that would completely answer what is asked in this task. Of course it is fine to use such references but please write in your own words and think of your own examples. Any books, webpages or other pieces of literature that you quote directly should be cited.