Pollution in the Great Lakes

Hannah Conroy Broderick   Charlene Gildea
Róisín Hill   Damian O'Connor

2nd Annual Stokes Modelling Workshop

June 18, 2015

Description of the problem

- The great lakes are the main water supply for 30 million people.
- Pollutants in the lakes are the main killer of marine life, encourage the growth of algae and cause obnoxious smells.
- The only feasible solution is to rely on the natural clean-up of the lakes.

- The task set was to create a mathematical model of the clean-up process to aid policy decisions.

Basis for single lake model

- Calculated for one lake
- Our assumptions
  - Constant volume in the lake
  - Uniform distribution of pollutant
  - Inflow of pollutant is constant over time

Single lake model calculations

$$\frac{d(PV)}{dt} = R_i P_i - R_o P_o$$

$P = P(t)$ - density of pollutant lake at time t
R_i/R_o - rate of inflow/outflow
P_i/P_o - density of pollutant in inflow/outflow
V - volume

$$P_o = P; R_i = R_o$$

$$V\frac{dP}{dt} + P\frac{dV}{dt} = R_i P_i - R_o P$$

$$\frac{dV}{dt} = R_i - R_o$$

$$V\frac{dP}{dt} = R_i(P_i - P)$$

Single lake model and solution

$$\frac{dP}{dt} = \frac{R_i(P_i - P)}{V};\ \frac{R_i}{V} = \tau$$

τ - $\frac{\text{Volume}}{\text{Rate of flow}}$, retention time

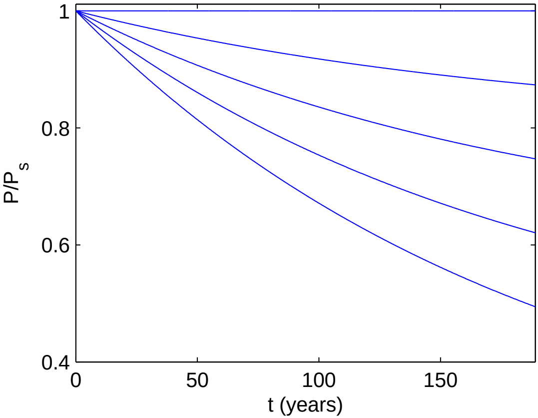
$$\frac{dP}{dt} + \frac{P}{\tau} = \frac{P_i}{\tau}$$

Solution

$$P(t) = P_i + (P_s - P_i)e^{\frac{-t}{\tau}}$$

P_s - initial density of pollutant in lake

Graph of First Model $P(t)/P_s$



Pollution of Lake Superior

Variation for single lake model

- Calculated for one lake
- Our assumptions
  - Constant volume in the lake
  - Uniform distribution of pollutant
  - Inflow of pollutant is reducing

Variation of single lake model

Model

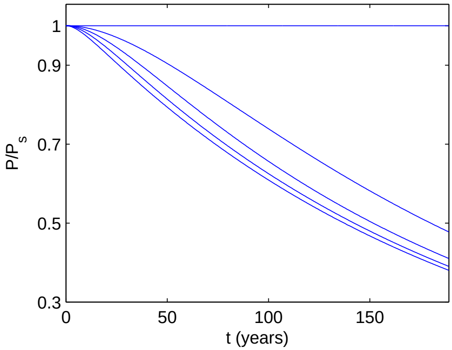
$$\frac{dP}{dt} + \frac{P}{\tau} = \frac{P_s e^{-at}}{\tau}; \text{ where } P_i = P_s e^{-at}$$

Solution

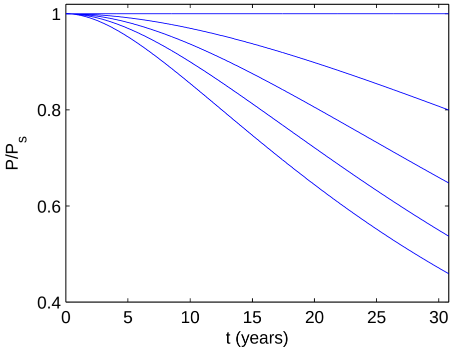
$$P(t) = \frac{P_s(a\tau e^{\frac{-t}{\tau}} - e^{-at})}{a\tau - 1}$$

$P(t)$ - density of pollutant lake at time t
P_s - initial density of pollutant in lake
τ - $\frac{\text{Volume}}{\text{Rate of flow}}$
a - $\frac{ln(\frac{P_s}{P_n})}{\text{planned years to achieve reduction}}$

Graph of variation of first model

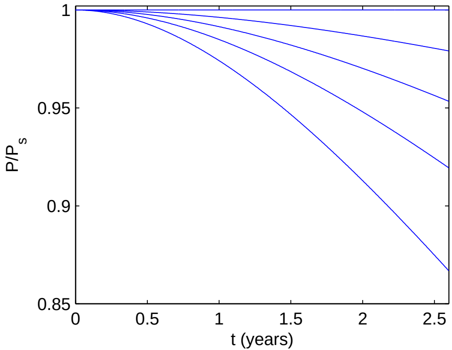


Lake Superior

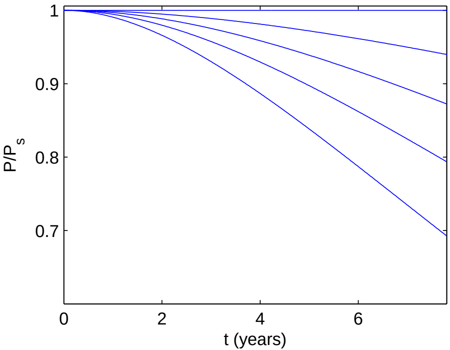


Lake Michigan/Huron

Graph of variation of first model



Lake Erie

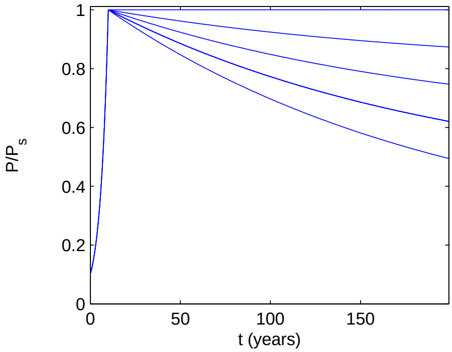


Lake Ontario

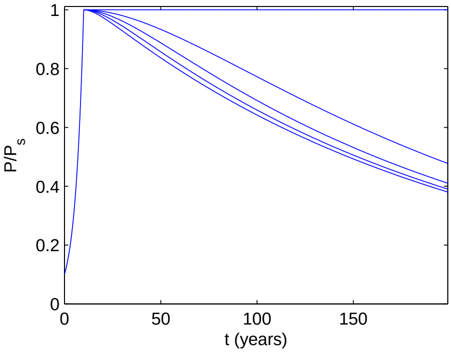
Initial exponential increase of pollutant

- Calculated for one lake
- Our assumptions
  - Constant volume in the lake
  - Uniform distribution of pollutant
  - Initial increase in pollutant
  - At a certain time pollutant level decreases

Initial exponential increase in pollutant



Constant input



Decreasing input

Basis for final models

- Calculated for two to four lakes, (Michigan and Huron as treated as one lake as they both are the same height above sea level)
- Our assumptions
  - Constant volume in each lake
  - Uniform distribution of pollutant
  - Inflow of external pollutant is constant
  - the direction of flow between the lakes is consistant with their height above sea level ie: Superior to Michigan/Huron to Erie to Ontario and finally into the Atlantic

Final model

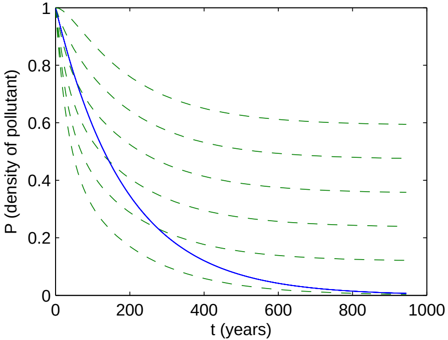
Final Model

$$\frac{dP_1}{dt} + \frac{P_1}{\tau_1} = \frac{P_{i1}}{\tau_1}$$

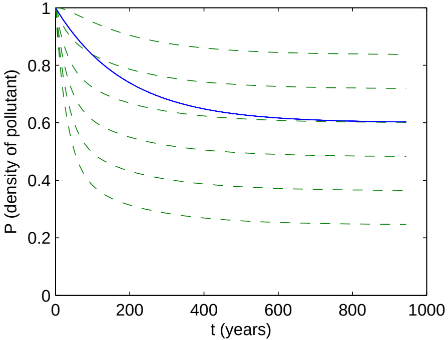
$$\frac{dP_2}{dt} + \frac{P_2}{\tau_2} = \frac{V_1}{V_2}\frac{(P_1 - P_{i2})}{\tau_1} + \frac{P_{i2}}{\tau_2}$$

P_1 - density of pollutant in Lake Superior
P_2 - density of pollutant in Lake Michigan/Huron
P_{i1}- density of pollutant in inflow to Lake Superior
τ - $\frac{\text{Volume}}{\text{Rate of flow}}$

Graph of Superior and Michigan/Huron

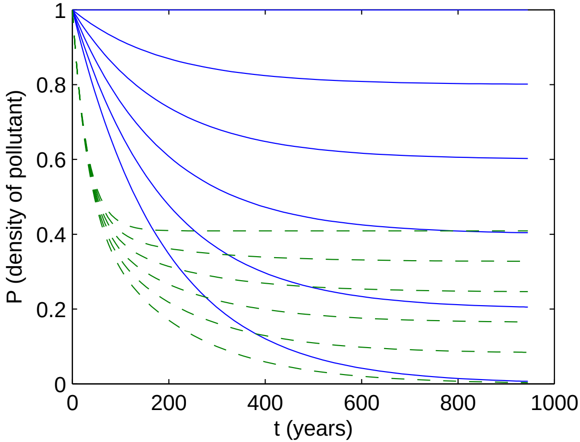


Input into Superior = 0



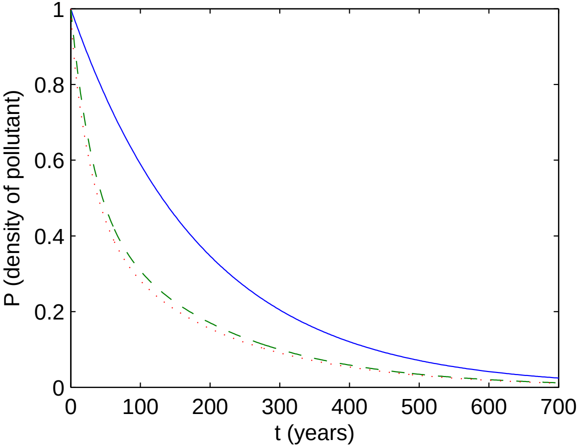
Input into Superior = 0.6

Graph of Two Lakes; Superior and Michigan/Huron



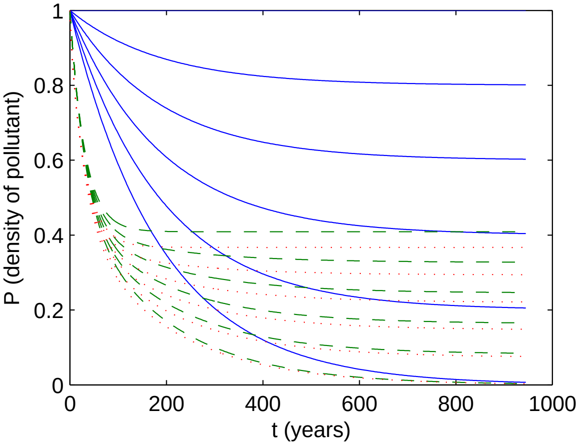
Varying Superior's input and constant input into Michigan/Huron

Graph of Three Lakes; Superior, Michigan/Huron and Erie



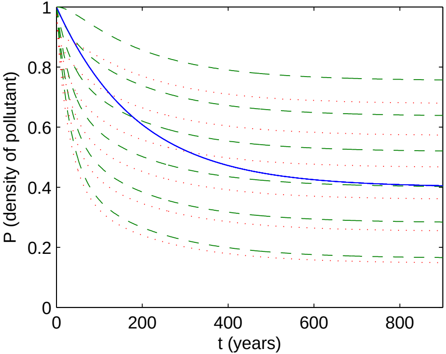
No input of pollutant

Graph of Three Lakes; Superior, Michigan/Huron and Erie

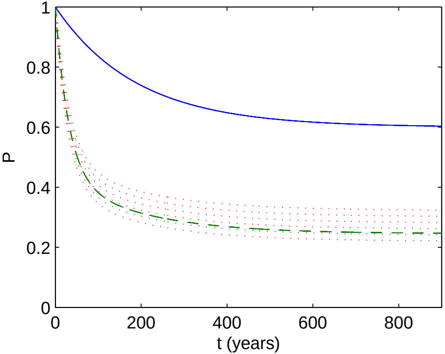


Varying Superior's input, no input to others

Graph of Three Lakes; Superior, Michigan/Huron and Erie

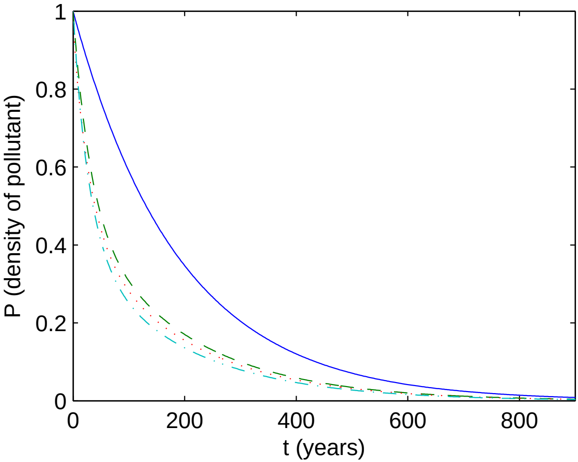


Varying Michigan/Huron only



Varying Erie only

Graph of Four Lakes



No input of pollutant

Conclusion

- The pollutant level tends to the input pollutant level when the input level is constant.
- Rate of outflow is constant so the levels of pollutants in the lakes will ultimately be determined by the level of input pollutants.

Further areas to investigate

- Varying constant inputs for multiple lake model
- Exponential decrease of pollution input for multiple lake model
- Diffusion of pollutants in the lakes
- The effect of a sudden spike in pollutants