Third Annual

Stokes Modelling Workshop

National University of Ireland, Galway

13-16 June 2016

Stokes Applied Mathematics Cluster
School of Mathematics, Statistics and Applied Mathematics
and
NUI Galway SIAM Student Chapter

Organising Committee:

Michel Destrade, Paul Greaney, Niall Madden, Robert Mangan
Petri Piiroinen, Eoghan Staunton, Michael Welby

List of Participants

Name	Institution
Jack Collins	NUI Galway
David Colson	NUI Galway
John Cormican	NUI Galway
Patrick Fleming	NUI Galway
Padraic Flood	NUI Galway
Sean Hehir	NUI Galway
Aoife Hill	NUI Galway
Róisín Hill	NUI Galway
John McFadden	NUI Galway
Claire Moran	NUI Galway
Cormac Murphy	Dublin Institute of Technology
Damian O'Connor	University of Limerick
Graham O'Connor	NUI Galway
Brian Regan	NUI Galway
Ashley Sheil	Maynooth University
Bram Siebert	NUI Galway
Szymon Urbas	NUI Galway
Andrew Whelan	Dublin Institute of Technology

Organising/Mentoring Team

Richard Burke
Michel Destrade
Paul Greaney
Niall Madden
Robert Mangan
Petri Piiroinen
Eoghan Staunton
Michael Welby
Giuseppe Zurlo

Timetable

	Mon 13/6	Tue 14/6	Wed 15/6	Thu 16/6
09:00		Work on Problems 09:00 - 10:00	Work on Problems 09:00 - 10:30	Work on Problems 09:00 - 12:00
09:30	Registration 09:30 - 10:00			
10:00	Welcome / Problem Presentations @ ADB-1020 10:00 - 11:00	Introduction to Matlab @ ADB-G021A 10:00 - 11:00		
10:30			Coffee, ADB-G021 10:30 - 11:00	
11:00	Coffee, ADB-G021 11:00 - 11:30	Coffee, ADB-G021 11:00 - 11:30	Work on Problems 11:00 - 13:00	
11:30	Group Allocation / Meet with Mentors @ ADB-1020 11:30 - 13:00	Introduction to Maple @ ADB-G021A 11:30 - 12:30		
12:00				Presentation of Results @ THB-G011, Hardiman Building 12:00 - 13:00
12:30		Work on Problems 12:30 - 13:00		
13:00	Lunch @ Friars Restaurant 13:00 - 14:00	Lunch @ Friars Restaurant 13:00 - 14:00	Lunch @ Bialann 13:00 - 14:00	Lunch @ Friars Restaurant 13:00 - 14:00
14:00	Work on Problems 14:00 - 17:30	Work on Problems 14:00 - 16:00	Work on Problems 14:00 - 16:00	Q&A on Life as a PhD Student, THB-G011, & Coffee 14:00 - 15:00
16:00		Presentation Skills Talk @ ADB-1020	Talk - Prof. Alfredo Marzocchi @ Plant Science Seminar Room, ADB-2018 16:00 - 17:00	
16:30		Work on Problems 16:30 - 17:30		
17:00			Work on Problems 17:00 - 18:00	
18:00			Social Event @ ADB-G021 18:00 - 20:00	

Problem Descriptions

1 Spread of Infectious Diseases in Human Populations

Skills required: Differential equations, numerical methods, some computational software (Matlab, Mathematica, R, C, Fortran, etc.)

Background. An *infection* is the invasion of an organism's body tissues by disease-causing agents and the reaction of host tissues to these organisms. An *infectious disease* is the illness resulting from an infection. Infections are typically caused by infectious agents such as viruses, viroids, prions, bacteria, nematodes, arthropods, fungi and other macroparasites. Hosts can fight infections using their immune system. Mammalian hosts react to infections with an innate response, often involving inflammation, followed by an adaptive response. Specific medications used to treat infections include antibiotics, antivirals, antifungals, antiprotozoals, and antihelminthics. Infectious diseases resulted in 9.2 million deaths in 2013 (about 17% of all deaths) (Source: Wikipedia).

Problem. Consider a population living on an island in which an infectious disease can spread. Build one or more mathematical models that can predict the number of individuals that may be infected or even die due to the disease. There are a number of possible scenarios that you can consider, for instance, the possibility of immunization through vaccination and the cost and benefit of this, different ways (direct contact, drinking water, mosquitoes etc.) in which the disease can spread and how isolated populations may be affected differently than a well-mixed population.

One of the most common mathematical models that deals with the spread of infectious diseases is the SIR model, which is framework based on compartments where each compartment represents a sub-group of the population. The model is typically made up of a system of nonlinear ordinary differential equations and normally S stands for the Susceptible group, I stands for the Infected group and R stands for the Recovered group. Since the SIR framework is very flexible it is easy to alter or extend it to encompass specific situations and can thus be used to answer specific questions.

There is plenty of information about infections diseases and SIR models in the literature. The task for the group working on this problem is to define a set specific problems of varied complexity that can be formulated mathematically and solved analytically and/or numerically.

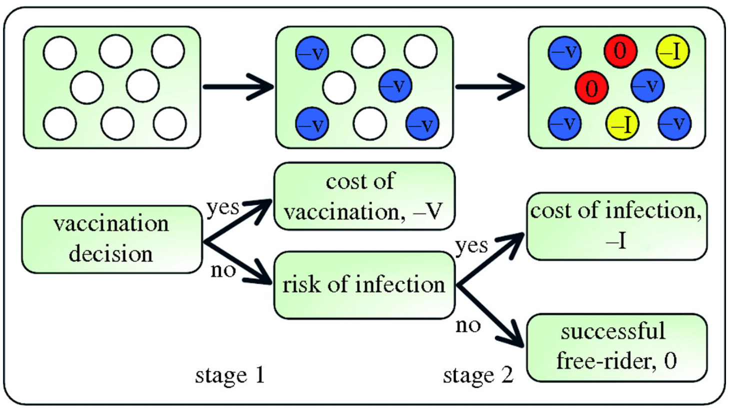

Figure 1: Schematic of a decision model for vaccination. Figure from: Feng Fu, Daniel I. Rosenbloom, Long Wang, Martin A. Nowak, *Imitation dynamics of vaccination behaviour on social networks*, Proceedings of the Royal Society B 278, pp. 42–49, July 2010. DOI: 10.1098/rspb.2010.1107

2 Playing with Elasticity

Skills required: Basic mechanics and elasticity, ordinary and partial differential equations, numerical analysis, equations of vibrating strings and beams.

A remarkable guitarist (portrayed below, left), after finishing his musical studies and having reached the hall of fame, decides to engage with yet another hurdle: to study Applied Mathematics. One day, while relaxing after study with a guitar in his hands, he starts wondering about questions that a famous Italian luthier from Cremona, Antonio Stradivari (portrayed below, right), had answered a long time before him with fantastic intuitions: how do the vibrations of a string reach our ears? What is the role of the *soundboard* (the holed piece of wood) and of the *bridge* (the device that supports the strings, close to our guitar-hero's right hand)? Presumably, the same basic mechanisms hold for all *stringed instruments* (piano, violin, guitar, ukulele. . .).

Unfortunately, Stradivari's secrets were buried with him. For this reason, nowadays (outrageously rich) musicians from all over the world prefer spending amazing amounts of money to get his original instruments than buying new ones (the *Lady Blunt* violin from Stradivari was recently sold for $15,984,000). So, as you may imagine, a rational unveiling of Stradivari's secrets may end up being very rewarding. . .


Your tasks in this project are:

1. first of all, we need to have a very simple, *one-dimensional* model of the string-bridge-soundboard system. The elastic behaviour of all elements, their mass and their damping must be accounted for. This should end up with a system of ODEs that we shall solve analytically (or numerically). Can we associate sounds, together with plots, to understand what happens in each item?

2. the second step is very hard. How are string vibrations *actually* transmitted to the soundboard across the bridge? Here we shall turn to a *two-dimensional* model. We shall try to understand the interaction between string and beam vibrations, in particular trying to understand what happens in the bridge! This should end up with a system of PDEs, that we may try solving and. . . animating to create a movie!

3. after having successfully accomplished with this project, you may start your own violin brand and. . . become rich!

3 Modelling Fort McMurray

Skills required: Differential equations (probably) and numerical analysis (definitely)

On 1 May 2016, a wildfire broke out about 15 km from Fort McMurray, an oil town in northern Alberta with a population of over 60,000. Within two days, it had grown to engulf part of the town, destroying several thousand buildings, and forcing a mass evacuation. Figure 2 shows composite images produced at the NASA Earth Observatory demonstrating the development of the fire over one day. The fire is still burning.

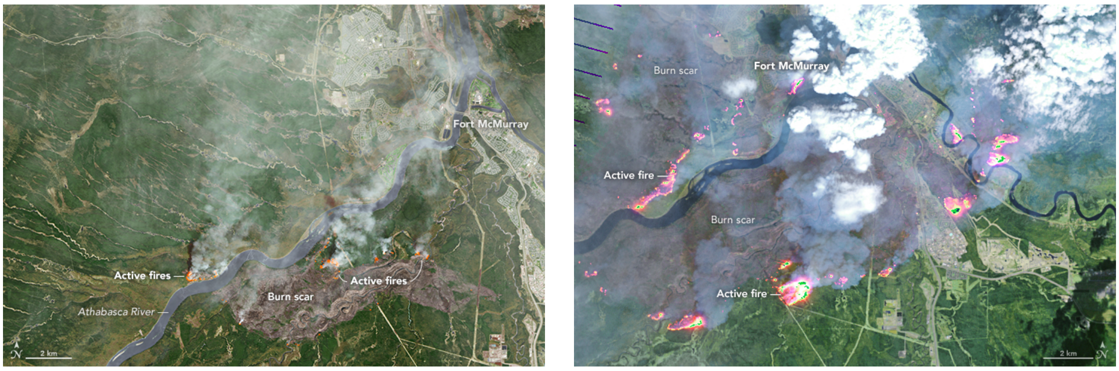

Figure 2: Satellite images of the Fort McMurray Fire on 3 May (left) and 4 May (right), from NASA Landsat project. Credit: NASA Earth Observatory

There were numerous contributing factors to the rapid growth of the fire, including the unusually dry winter, and unseasonably high temperatures at the time of the outbreak.

Could the spread of the fire have been predicted? Your job is to develop a model of the spread of a wildfire, taking into account factors like wind, air temperature, precipitation, etc. Then this model should be used to simulate (numerically) the spread of the fire, providing a tool that can be used by those who need to plan evacuations, fire-fighting strategies, etc.

The team working on this project will

- devise a simple model for the spread of wildfire;
- locate suitable data to parameterize and validate the model;
- implement a suitable numerical scheme so the model can be used to generate simulations to obtain predictions;
- discuss how the model could be made more realistic and/or applied to realistic settings such as the Fort McMurray fire.

4 Who's the Bas?

Skills required: ODEs and PDEs, special functions, taste for numerics

I'm working for a French sports company looking to expand its activity to Ireland. From a quick search I found out that the market for hurley sticks is quite large (half a million units or so produced annually), and that there might be some room for improvement in the design. It seems all hurls are made from ash, except for one company making synthetic ones. The concept of a "sweet spot" also seems really important, especially now that penalties have to be taken from the 20m line.

I'd like to find out:

- Can the sweet spot area be related to the modes of vibration of the hurl's plate?
- Are those vibration modes affected by the addition of a metal plate, or of a resin coat, or of the orientation of the wood grain?
- Can a synthetic hurl be designed to take advantage of the knowledge on vibration modes?

To answer these questions we can take

- the analytic route: using simplified geometries and special functions to solve the plate equations exactly;
- the numerical route: using ABAQUS, MATLAB, MAPLE to simulate ball/bas striking events;
- a combination of both or more!

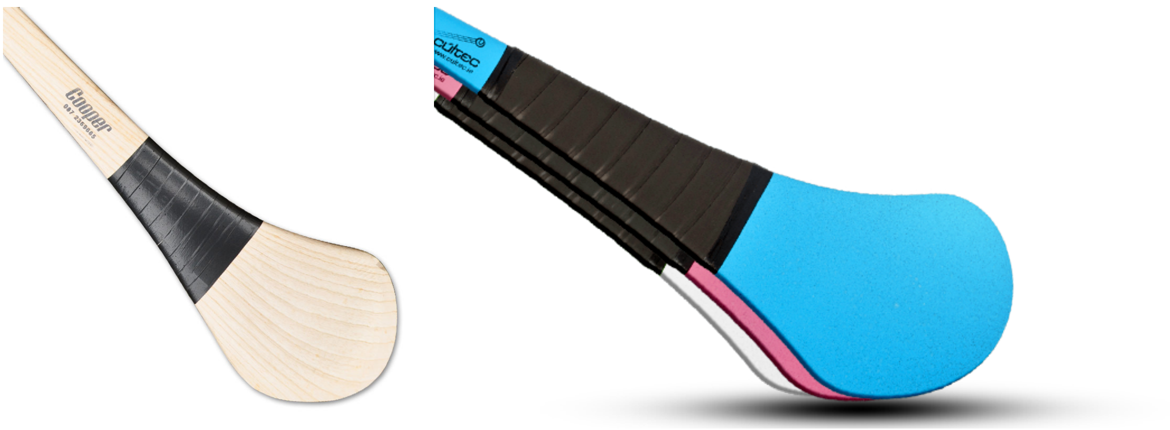

Figure 3: Hurls made of Ash tree vs synthetic hurls