Playing with elasticity

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NUI Galway
OÉ Gaillimh

Stokes CLUSTER
FOR APPLIED MATHEMATICS
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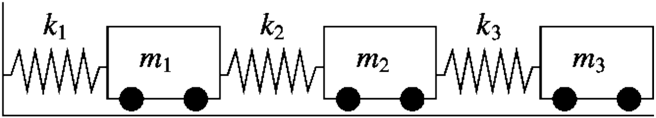
Introduction

We have conducted an investigation into the string-bridge-soundboard systems of acoustic instruments.

We created a simplified model using classical mechanics, and then used it to gain intuition for the forces at play in the system.

Basic Model

Our basic model is a three spring-mass system



m_1: soundboard, m_2: bridge, m_3: string.

Equations

Equations for the springs and masses

$$m_1\ddot{x}_1 + k_1x_1 + k_2(x_2 - x_1) - c_1x_1 + c_2(\dot{x}_2 - \dot{x}_1) = 0$$

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) - k_3(x_3 - x_2) - c_2(x_2 - x_1) + c_3(\dot{x}_3 - \dot{x}_2) = 0$$

$$m_3\ddot{x}_3 + k_3(x_3 - x_2) - c_3(\dot{x}_3 - \dot{x}_2) - F = 0$$

where c_1, c_2 and c_3 are the damping coefficients of springs 1,2, and 3 respectively.

Solution

Solved in the following form

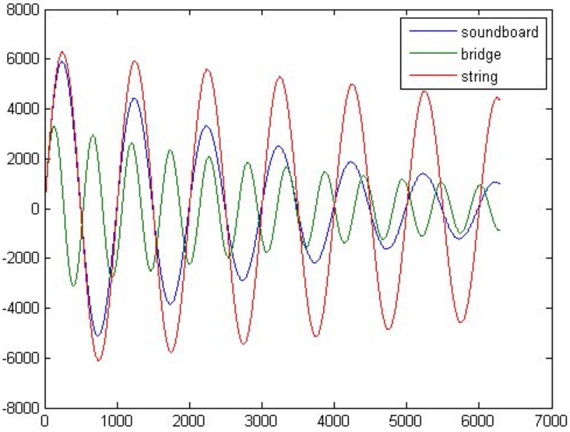
$$M\ddot{x} + C\dot{x} + Kx$$

With

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} C = \begin{bmatrix} -c_1 - c_2 & c_2 & 0 \\ c_2 & -c_2 - c_3 & c_3 \\ 0 & c_3 & -c_3 \end{bmatrix}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \ddot{x} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Sound Plot



8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
0
1000
2000
3000
4000
5000
6000
7000
soundboard
bridge
string

String, bridge and soundboard

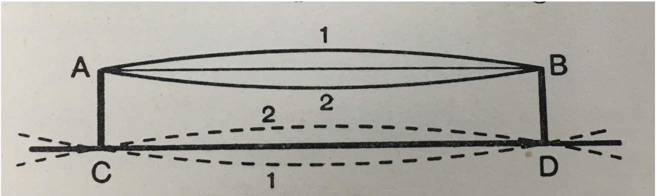
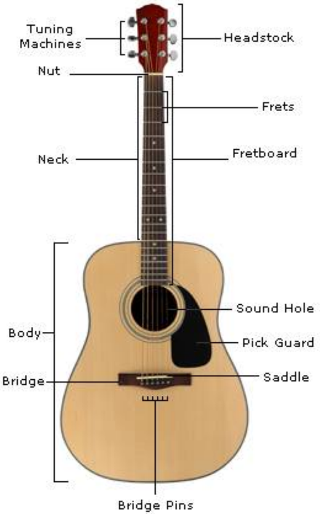
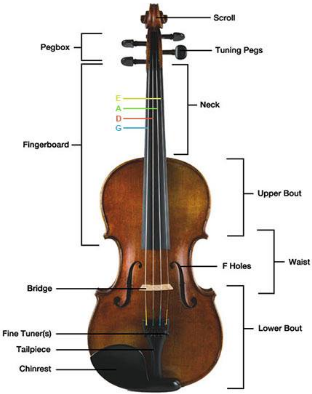


FIG. 30

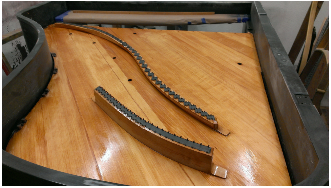


Guitar



violin

harp



piano bridge

References

A.B.Wood : A Text-book of Sound