Contents

Preface 2

Programme 3

Plenary speakers' abstracts 4

Student speakers' abstracts 7

Poster abstracts 20

Participant List 22

Preface

This is the second annual conference organised by the NUI Galway SIAM Student Chapter, since it was established in September 2014.

The main purpose of SIAM student chapters is to facilitate interdisciplinary collaboration between students from different disciplines, including mathematics, applied mathematics, statistics, computer science, engineering, economics, physics and other sciences.

This conference aims to bring students together from all over Ireland, (and beyond), whose main areas of research involve industrial or applied mathematics. This will give everyone involved the chance to gain an insight into the work being carried out by graduate student researchers across a range of institutions and disciplines. In doing so, we hope to foster networks of researchers across various universities, and encourage greater collaboration between disciplines.

Finally, from the organising committee, we hope that you find this meeting interesting and stimulating, and that you can join us for a few refreshments in the Christmas market afterwards. Have an enjoyable time here in Galway and we hope to see you all again soon.

Acknowledgements: Supported by SIAM, the Stokes Cluster (NUI Galway), MACSI (University of Limerick) and Complex Systems Research Centre (CORE).

Local Organising Committee:

Richard Burke (Chapter President)

Christine Marshall (Chapter Vice-President)

Paul Greaney (Chapter Treasurer/Secretary)

Niall Madden (Faculty Advisor)

Faiza Alssaedi

Shane Burns

Stephen Russell

Bram Siebert

Eoghan Staunton

Michael Welby

Programme

9.15	**Registration**	
9.45	Prof. Donal Leech *Dean of Science*	Opening Remarks
10.00	Prof. James Gleeson *University of Limerick*	Information contagion on social networks
10.40	Brendan Murray *University College Dublin*	Phase precession evolution in the Burgers equation on a fractal Fourier set
11.00	**Coffee Break**	
11.30	Naoise Holohan *Trinity College Dublin*	Extreme points of the differential privacy polytope
11.50	Niall McInerney *University of Limerick*	A moving boundary problem in controlled release pharmaceuticals
12.10	Martina Curran *National University of Ireland Galway*	An R-based framework for implementing large scale spatial models of infectious diseases
12.30	**Lunch and Poster Session**	
13.50	Dr Dana Mackey *Dublin Institute of Technology*	Car parking, immunosensors, and the dynamics of interdisciplinary collaborations
14.30	Gary O'Keeffe *University of Limerick*	Modelling the efficiency of a nanofluid-based solar collector
14.50	**Coffee Break**	
15.20	Alan Compelli *Dublin Institute of Technology*	The Canonical Hamiltonian structure of bounded rotational internal geophysical waves
15.40	Kevin Devine *University of Limerick*	Oscillation marks in the continuous casting of steel
16.00	Closing Remarks	

Information contagion on social networks

James Gleeson[†], **Kevin O'Sullivan**[†]
Raquel Banos[‡], **Yamir Moreno**[‡]

[†]MACSI, Department of Mathematics and Statistics
University of Limerick, Limerick, Ireland
james.gleeson@ul.ie

[‡] Universidad de Zaragoza

ABSTRACT

We consider the spreading of "memes" (distinct pieces of information like ideas, hashtags, URLs, etc.) on a large directed social network, like Twitter. We use a branching-process model to describe how users choose among multiple sources of incoming information, similar to models used in other studies, which rely on intensive computational simulations to fit to data. In contrast, we here develop analytical insights into the respective roles of the network degree distribution, the memory-time distribution of users, and the competition between memes for the limited resource of user attention. The result is a form of self-organised criticality, which we dub "competition-induced criticality". Using this analysis, we fit the model to data on Twitter hashtags, and predict features of the time-dependent data. This is joint work with Kevin O'Sullivan (UL), and Raquel Banos and Yamir Moreno (both Universidad de Zaragoza).

Car Parking, Immunosensors, and the Dynamics of Interdisciplinary Collaborations

Dana Mackey[†], **Eilis Kelly**[†]
Robert Nooney[‡], **Colette McDonagh**[‡]

[†]School of Mathematical Sciences,
Dublin Institute of Technology, Ireland
dana.mackey@dit.ie
eilis.kelly@dit.ie

[‡] Biomedical Diagnostics Institute
Dublin City University, Ireland

ABSTRACT

Monolayer particle deposition has been successfully described by the model known as *random sequential adsorbtion* (RSA). Rigid particles are placed at random, sequentially and irreversibly onto solid smooth surfaces in such a way that the particles do not overlap. If an incoming particle approaches an already covered part of the substrate, it is rejected. Eventually no more particles fit on the surface and the process stops in the so-called *jamming limit*. In one dimension, this process is commonly referred to as "the parking problem" and the jamming coverage, also known as the Renyi constant, has been calculated as $C_R \approx 0.74756....$

The motivation for this work is optimising fluorescent sandwich immunoassays, where capture and label antibodies are immobilized on planar or spherical substrates. Immunoassays are used to detect antigen markers for a variety of diseases including HIV, hepatitis, or cardiovascular disease with high specificity and sensitivity in a range of media including blood or urine. Antibodies are large molecules containing many parts, in which the position and accessibility of the antigen-biding site, located in the FAB region, depend on the orientation and packing of the antibody.

Using the RSA framework, we propose a few mathematical models in order to calculate how the concentration of correctly oriented molecules (active site exposed for subsequent reactions) evolves during the deposition process. It has been suggested by experimental studies that high concentrations will decrease the assay performance, due to molecule denaturation and obstruction of active binding sites. However, crowding of antibodies can also have the opposite effect by favouring upright molecule orientations. We therefore attempt to predict which of these competing effects prevails under different experimental conditions and find the coverage which yields the maximum expected concentration of active molecules (and hence the highest signal).

Acknowledgement. The second author is supported by an Irish Research Council postgraduate scholarship.

References

[1] Abaqus Analysis User's Manual, Simulia, `www.simulia.com`, 2011.

[2] G.I. Barenblatt and D.D. Joseph, eds., *Collected Papers of R.S. Rivlin*. Volume 1,2, Springer-Verlag, New-York, 1997.

[3] G.A. Holzapfel, T.C. Gasser and R.W. Ogden, A new constitutive framework for arterial wall mechanics and a comparative study of material models. *J. Elasticity* 61:1-48, 2000.

[4] G.A. Holzapfel, *Nonlinear Solid Mechanics. A Continuum Approach for Engineering*. John Wiley & Sons, Chichester, 2000.

[5] J.D. Humphrey, Need for a continuum biochemomechanical theory of soft tissue and cellular growth and remodeling, in: G.A. Holzapfel and R.W. Ogden, eds., *Biomechanical Modelling at the Molecular, Cellular and Tissue Levels*, Springer-Verlag, Wien, New York, CISM Courses and Lectures no. 508, pp. 1-82, 2009.

Phase and precession evolution in Burgers equation on a fractal Fourier set

Brendan Murray[†], **Miguel Bustamante**[‡]

Complex and Adaptive Systems Laboratory, School of Mathematics and Statistics,
University College Dublin, Belfield, Dublin 4, Ireland
[†] brendan.murray@ucdconnect.ie
[‡] miguel.bustamante@ucd.ie

ABSTRACT

This work is a study of the phase dynamics of the one-dimensional stochastically forced Burgers equation evolved on a fractal Fourier set. A connection is uncovered between coherent structures in real space and the evolution of triads in Fourier space. Real space structures are associated with entangled correlations amongst the phase precession frequencies and the amplitude evolution of triads in Fourier space. As a result [1], triad precession frequencies show a non-Gaussian distribution with multiple peaks and fat tails, and there is a significant correlation between triad precession frequencies and amplitude growth. By changing the fractal dimension D of the underlying Fourier set, we observe a depletion of intermittency as a function of the fractal dimension, and the simultaneous reduction of the correlation between the growth of Fourier mode amplitudes and the precession frequencies of triad phases. The stochastically forced Burgers equation in 1-D, given by eq. (1).

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u^2}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2} + f, \tag{1}$$

where $u = u(x,t) = \sum_{\mathbf{k}\in\mathbb{Z}} \hat{u}(t)\mathrm{e}^{\mathrm{i}kx}$ with Wavevector k.

To facilitate our analysis we decompose the solution field into Fourier modes, where each Fourier component $\hat{u}_k(t)$ satisfies the ODE evolution equation given below in eq. (2).

$$\frac{d\hat{u}_k}{dt} = -\frac{ik}{2}\sum_{k_1} \hat{u}_{k_1}\hat{u}_{k-k_1} - \nu k^2 \hat{u}_k + \hat{f}_k, \tag{2}$$

To further separate the variables of interest an amplitude/phase representation will be used, where $\hat{u}_k(t) = a_k(t)e^{i\phi_k(t)}$ with $a_k(t)$ the real valued amplitude component and the phase $\phi_k(t) = \arg[\hat{u}_k(t)]$. In Burgers equations the nonlinearity is quadratic so interactions appear in *triads* (groups of 3 Fourier modes). The *key* dynamical degrees of freedom consist of the modes' real amplitudes $a_k(t)$ along with the *triad phases*, i.e. the combinations:

$$\varphi^{k_3}_{k_1\,k_2}(t) = \phi_{k_1}(t) + \phi_{k_2}(t) - \phi_{k_3}(t),$$

where the wavenumbers k_1, k_2, k_3 satisfy a 'closed-triad' condition: $k_1 + k_2 = k_3$ The evolution of these triad phases $\varphi^{k_3}_{k_1\,k_2}(t)$ can then be explored, in particular how the probability density function changes with fractal dimension D and whether phase precession frequencies in forced Burgers can take non-zero values with significant probability, so that in future work we can apply the methods developed in [2] to trigger the precession resonances.

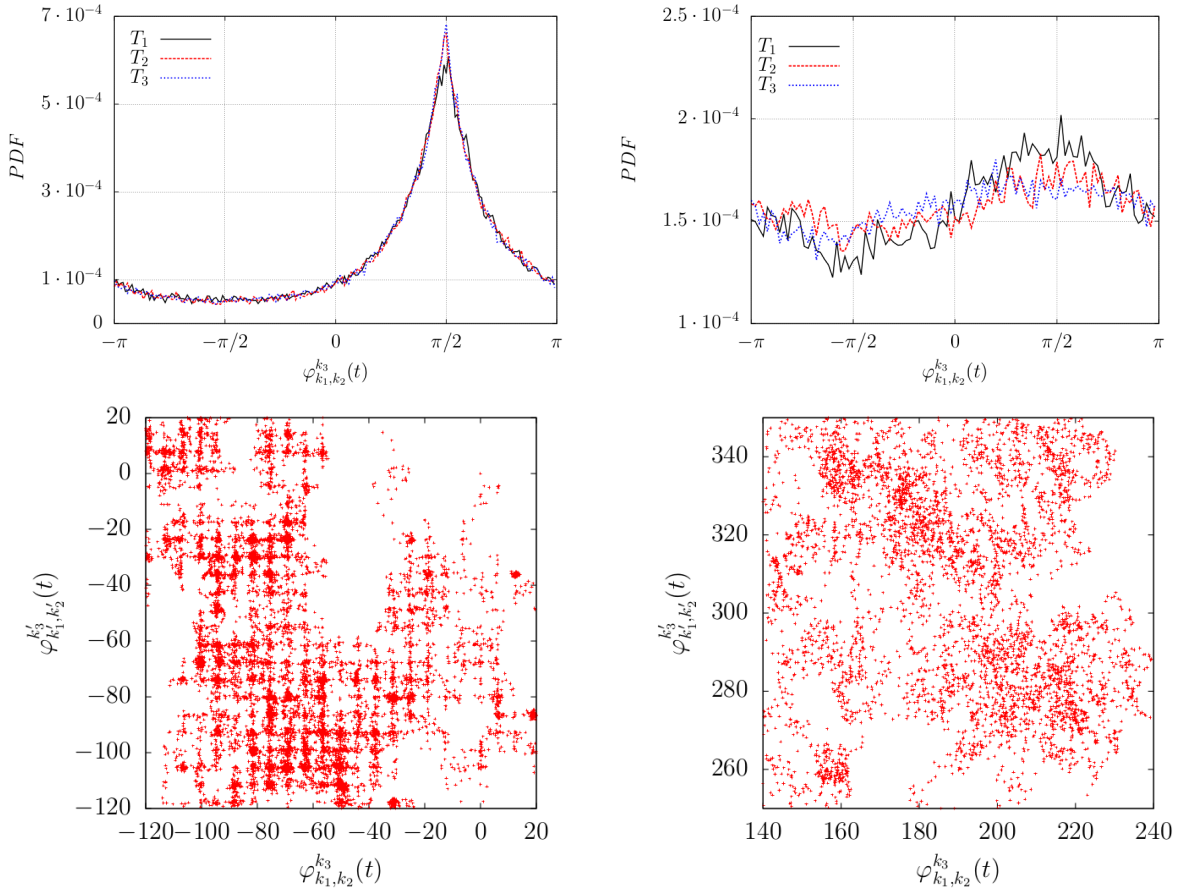

Figure 1: **Top**: Histograms of triad phase (mod 2 π) with offset $-\pi$ computed during the temporal evolution of different triads in the inertial range. $T_1 : [k_1; k_2; k_3] = [100; 150; 250]$ (black solid line), $T_2 : [k_1; k_2; k_3] = [200; 250; 450]$ (red dashed line), and $T_3 : [k_1; k_2; k_3] = [300; 350; 650]$ (blue dotted line). *Left*: Undecimated case: Fractal Fourier dimension $D = 1$. *Right*: $D = 0.95$. **Bottom**: Parametric plot of the phase evolution for two different triads in the inertial range. $T_1 : [k_1; k_2; k_3] = [100; 150; 250]$ and $T_2 : [k_1; k_2; k_3] = [200; 250; 450]$ showing synchronisation events near $\frac{\pi}{2} + 2n\pi$, $n \in \mathbb{Z}$. *Left*: Undecimated case: Fractal Fourier dimension $D = 1$. *Right*: $D = 0.95$.

References

[1] M. Buzzicotti, B. P. Murray, L. Biferale & M. D. Bustamante , Phase and precession evolution in the Burgers equation. Submitted to The European Physical Journal E, *arXiv:1509.04450*, 2015.

[2] M. D. Bustamante, B. Quinn, & D. Lucas , Robust Energy Transfer Mechanism via Precession Resonance in Nonlinear Turbulent Wave Systems. *Physical Review Letters*, 113(8), 084502. doi:10.1103/PhysRevLett.113.084502, 2014.

Extreme Points of the Differential Privacy Polytope

Naoise Holohan[†], **Douglas J. Leith**[†], **Oliver Mason**[‡]

[†]School of Computer Science and Statistics, Trinity College Dublin
nholohan@tcd.ie

[‡]Hamilton Institute, Maynooth University

ABSTRACT

Data privacy concerns the release of private information for commercial/scientific research, but doing so in a way that preserves the privacy of the participants. Although researched for decades, interest in it has accelerated since the turn of the millennium. Fields such as computer science, cryptography and law have led the drive, but data privacy also presents exciting new research opportunities in mathematics.

We consider data perturbation in the local privacy setting, whereby subjects perturb/sanitise their own data locally, before supplying it to a central database where queries are answered. If the database seeks for participants to contribute one value from the list $\{val_1, \ldots, val_n\}$ (i.e. blood type), everyone provides their data through a mechanism which randomly selects an output for each input:

$$\mathbb{P}(\text{output} = val_j | \text{input} = val_i) = a_{ij}. \tag{3}$$

The mechanism is determined by $A \in \mathbb{R}^{n \times n}$, which must be stochastic and non-negative.

One model of privacy that has gained considerable traction in the last decade is differential privacy, defined using a privacy parameter $\epsilon \geq 0$. For local privacy, it requires

$$a_{ij} \leq e^{\epsilon} a_{kj}, \tag{4}$$

for each i, j, k. All matrices $A \in \mathbb{R}^{n \times n}$ that satisfy local ϵ-differential privacy (and are stochastic and non-negative), define a convex polytope in $\mathbb{R}^{n \times n}$.

In this talk we will examine this novel polytope and, more specifically, its extreme points. As the polytope is bounded, its extreme points can effectively tell us all we need to know. What form do they take, what important properties do they have, and how can we discover/create them?

The extreme points are also useful when considering optimal mechanisms. Given a linear utility function, the mechanism(s) which maximise utility (i.e. give the most useful answers) will lie at an extreme point. It is therefore in our interest to know how they behave.

A Moving Boundary Problem In Controlled Release Pharmaceuticals

Niall McInerney[†], **Dr. Sarah Mitchell**[‡]

[†]MACSI, Department of Mathematics and Statistics
University of Limerick, Limerick, Ireland
niall.mcinerney@ul.ie

[‡]MACSI, Department of Mathematics and Statistics
University of Limerick, Limerick, Ireland
sarah.mitchell@ul.ie

ABSTRACT

Polymer based controlled drug delivery devices are becoming increasingly popular in the pharmaceutical industry, due to the ability of chemists to synthesise polymers with desired properties to suit a required release profile. This can alleviate the disadvantages of traditional drug delivery, such as inaccurate dosages and patient compliance, minimising toxicity and maximising effectiveness. Mathematical modelling is an important tool in facilitating product development in the pharmaceutical industry, assisting is determining the geometries and configurations of these devices, as well as the desired drug loading concentrations.

Whilst there has been extensive studies of drug release mechanisms both from a theoretical and experimental point of view, there is little research into fully understanding the moving interface between the dissolved and loaded drug, especially in the case of swelling polymers where the polymer changes from a fully swollen to a fully collapsed state upon contact with an environmental fluid. In swelling controlled devices, as seen in Fig. 1, the polymer matrix is initially in a dry glassy state with the drug molecules dispersed and unable to diffuse. The polymer swells upon contact with an environmental fluid (solvent), which then diffuses into the polymer (often hydrophilic hydrogels), creating a moving boundary separating the dry polymer from the now swollen rubbery polymer, within which the drug can now diffuse. There is also a second moving boundary at the edge of the polymer as a result of volume change. There is a vast scope for developing simple approximate solutions which capture the key features of the model but are tractable and easy to implement.

We revisit the work of Cohen and Erneux [2], who proposed a coupling of the popular Higuchi [3] model describing drug release with the Astarita and Sarti [1] model describing a solvent penetrating a polymer in one dimension. This Stefan problem then has two distinct sections, the first describes the concentration of the solvent $C(X,T)$ that diffuses into the device and the position of the two moving boundaries $S_1(T)$ moving into the polymer and $S_2(T)$ moving outwards because of the volume change. The solutions for $S_1(T)$ and $S_2(T)$ are then used as an input into the second section of the model describing the countercurrent diffusion of the drug with concentration $A(X,T)$.

We suggest a different scaling to Cohen and Erneux, recovering three non-dimensional parameters in the process, all of whose size can vary. We then extend their analysis by doing small and large time asymptotics, but these are limited due to the parameter sizes needing to be $O(1)$. We then apply the popular heat balance integral method (HBIM) and refined integral method (RIM), and describe how to optimise the exponents of the approximating polynomials for both the solvent and drug. The results are shown to be accurate for all time

and varied parameter values. A numerical solution will also be discussed, using the implicit, unconditionally stable and second-order accurate Crank-Nicolson scheme. Finally, I will briefly describe a nonlinear diffusion problem, where the diffusion coefficient for the drug depends on the concentration of the solvent, as well as outline future research.

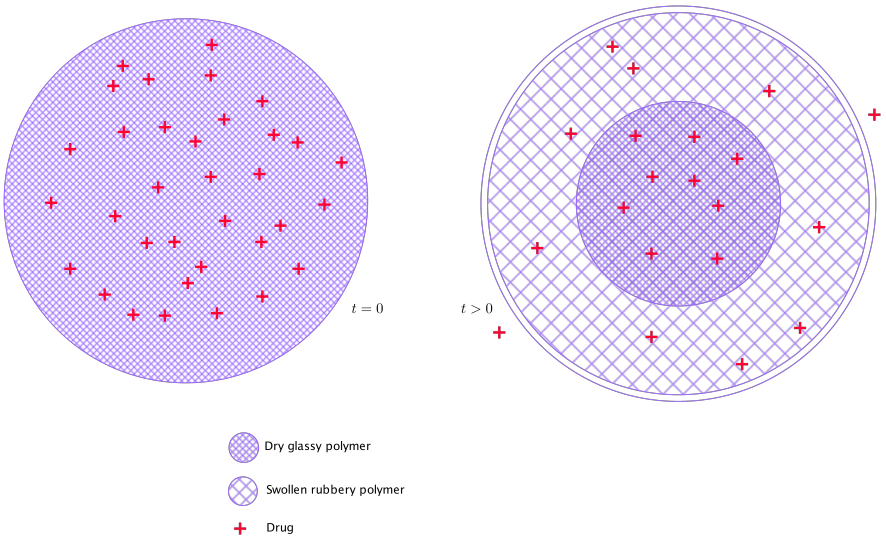

Figure 1: Schematic of a swelling polymer.

References

[1] G. Astarita and G.C. Sarti. A class of mathematical models for sorption of swelling solvents in glassy polymers. *Polymer Engineering & Science,* 18(5):388-395, 1978.

[2] D.S. Cohen and T. Erneux, Free boundary problems in controlled release pharmaceuticals: II. swelling-controlled release. *SIAM Journal on Applied Mathematics,* 48(6):1466-1474,1988.

[3] T. Higuchi, Mechanism of sustained-action medication. theoretical analysis of rate of release of solid drugs dispersed in solid matrices. *Journal of pharmaceutical sciences,* 52(12):11451149, 1963.

An R-Based Framework for Implementing Large-Scale Spatial Models of Infectious Diseases

Martina Curran[†], **Enda Howley**[‡], **Jim Duggan**[†]

[†]College of Engineering and Informatics
National University of Ireland Galway, University Road, Galway, Ireland
m.curran8@nuigalway.ie

[‡]College of Engineering and Informatics
National University of Ireland Galway, University Road, Galway, Ireland
e.howley@nuigalway.ie

[†]College of Engineering and Informatics
National University of Ireland Galway, University Road, Galway, Ireland
jim.duggan@nuigalway.ie

Abstract

Seasonal Influenza is a problem globally, and is estimated to result in about 3 to 5 million cases of severe illness, and about 250,000 to 500,000 deaths [2] annually. Modelling infectious diseases is an essential tool when faced with epidemics, as early detection is crucial in order to predict infection dynamics.

The current challenge in the epidemiology field is fast response times: trying to tell when and where to best direct attention and actions before the spread becomes too large to manage. These models can help in making decisions on whether or not herd immunity is possible at any stage of an epidemic, and if so, at what level. This creates a need for a lower-level, detailed mathematical model to measure the spread of infections.

For highly-transmittable infectious diseases such as influenza, the travelling patterns of individuals play an essential role in geographical spread [1]. In order to see the geographical spread of an infection through a population, we break it down into sub-populations, which allows us to see how the factors from the single population model impact on each other in a continuous manner: ensuring a more accurate prediction, allowing for faster response times, and more directed interventions.

The programming language R was used to develop a model which can be used as a decision support system by epidemiologists, giving a better understanding of different infection scenarios. It can be used as a framework to model the spread of infections at a spatial level, and is scalable for any size and geography of population.

The case study carried out uses Irelands national census data for the 40 electoral constituencies. This disaggregated model allows visualisation of the change in stocks for any number of sub-populations, with a clearer idea of the infections in each single population, and the peak of the individual infection curves, as can be seen in Fig. 1.

However, with increasing population numbers, it becomes difficult to visually separate the constituecies in the graph. Using the shape files also allows the retrieval of coordinate data from the Census, and using the package maptools, gives a method of visually mapping the spread within each electoral constituency at the same time as can be seen in Fig. 2. Using shape files with the model allows it to be used for any number of populations, and any country or continent. It can be broken down into different spatial cohorts, depending on the shape files available. This allows a simplified use of the model, as all details regarding the populations (population names, population numbers, and area) are often included in shape files.

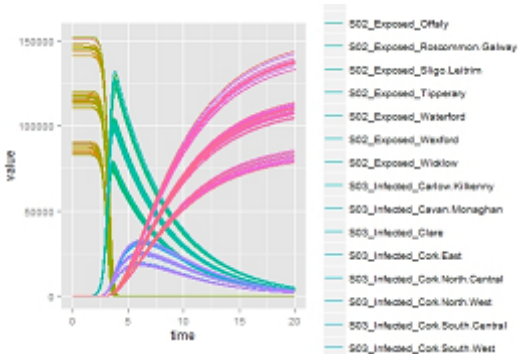

Figure 1: Disaggregated SEIR Model for Irish Constituencies

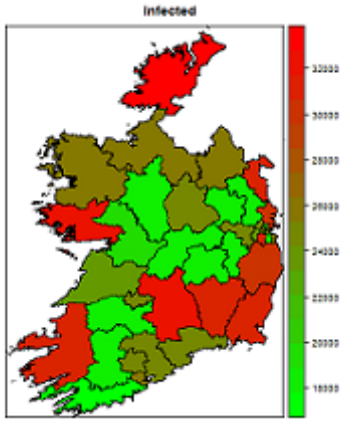

Figure 2: Number of Infectious People in each Constituency

References

[1] Chong, K.C., Ying Zee, B.C., Modeling the impact of air, sea, and land travel restrictions supplemented by other interventions on the emergence of a new influenza pandemic virus. *BMC Infectious Diseases* 12:309, 2012.

[2] Influenza (Seasonal) Fact Sheet, WHO, `www.WHO.int`, 2014.

Modelling the Efficiency of a Nanofluid-based Solar Collector

Gary O'Keeffe[†], Dr. Sarah Mitchell[†], Prof. Tim G. Myers[‡], Dr. Vincent Cregan[‡]

[†]Department of Mathematics and Statistics
University of Limerick, Castletroy, Limerick, Ireland
Gary.OKeeffe@ul.ie
Sarah.Mitchell@ul.ie

[‡]Centre de Recerca Matemàtica
Campus de Bellaterra, Edifici C, 08193 Bellaterra, Barcelona, Spain
tmyers@crm.cat
vcregan@crm.cat

ABSTRACT

It is estimated that 30 min of solar radiation falling on earth is equal to the world energy demand for one year [2]. Unlike traditional direct absorption solar collectors (DASC's), Nanofluid based Direct absorption solar collectors (NDASC's) use a colloidal solution of nanoparticles and a base fluid to absorb incident sunlight. This method increases efficiency over flat plate solar collectors and DASC's [3, 4]. The actual theoretical increase in efficiency is unclear and depends on the mathematical assumptions made while creating a model.

In this talk we present an approximate analytical solution to the steady state, two-dimensional model for the efficiency of an inclined nanofluid-based direct absorption solar collector. A schematic of the problem is depicted in Fig. 1.

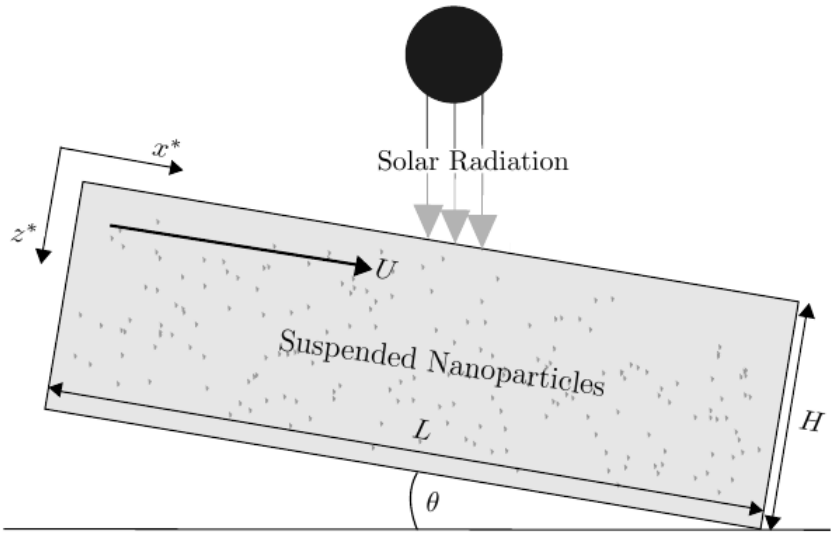

Figure 1: Schematic of an inclined nanofluid-based direct absorption solar collector of length L and height H.

The model consists of a system of partial differential equations describing the conservation of mass, momentum and energy, in the particular case of nanofluids moving within a solar energy installation. By applying the method of separation of variables to the equations describing the conservation of mass we find a depth dependent velocity profile in the nanofluid. A heat

source term is obtained via the radial flux integral, which is highly non-linear with respect to wavelength due to the spectral-dependent fluid and nanoparticle indices of refraction and absorption. To make analytical progress we introduce an approximate power-law function for the radial flux. This approximation of the integral can also be found in [1]. Applying the method of separation of variables, the resulting solution is used to investigate the efficiency of the collector subject to variation in model parameters.

We assume the velocity of the nanofluid in the solar collector is depth dependent while [1] assumes the velocity is constant. In spite of this added level of complexity, we are still able to solve the problem analytically. We also discuss the extension to a three dimensional model in a cylindrical pipe using cylindrical geometry. This approach adds considerable complexity, and the problem must be solved numerically.

Acknowledgement. We would like to acknowledge the Irish Research Council who are funding this research.

References

[1] V. Cregan, and T.G. Myers, Modelling the efficiency of a nanofluid direct absorption solar collector. *International Journal of Heat and Mass Transfer* 90:505-514, 2015.

[2] S.A. Kalogirou, Solar thermal collectors and applications. *Progress in energy and combustion science* 30:231-295, 2004.

[3] A.W. Lenert, and E.N. Wang, Optimization of nanofluid volumetric receivers for solar thermal energy conversion. *Solar Energy* 86:253-265, 2012.

[4] T.P. Otanicar, P.E. Phelan, R.S. Prasher, G. Rosengarten, and R.A. Taylor, Nanofluid-based direct absorption solar collector. *Journal of renewable and sustainable energy* 2:033-102, 2010.

The Canonical Hamiltonian Structure of Bounded Rotational Internal Geophysical Waves

Alan Compelli[†]

[†]School of Mathematical Sciences, College of Sciences & Health,
Dublin Institute of Technology, Kevin Street, Dublin 8, Ireland.
alan.compelli@dit.ie

ABSTRACT

This study aims to follow on from previous studies of internal waves in irrotational [4] and rotational [1] systems by including both non-zero vorticity and the implications of solid body rotation on the Hamiltonian formulation of a fluid system with lower and upper boundaries.

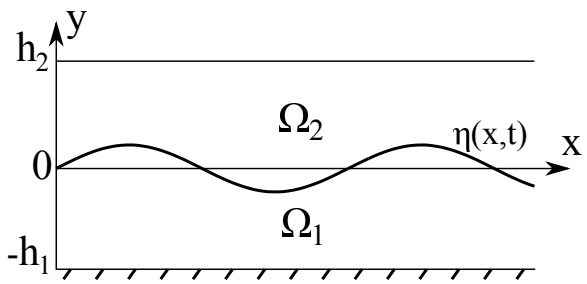

Figure 1: System setup.

An oceanic system is studied comprising of 2 domains, Ω_1 and Ω_2, as per Fig. 1. The domains are bounded underneath by a *flatbed* and above by a *lid* (assuming any surface waves have negligible amplitude). The subscript $i = \{1, 2\}$ represents evaluation in each respective domain. An internal wave acts as a free common interface between the media. $\eta(x, t)$ describes the elevation of the internal wave with evaluation at the internal wave given by the subscript c. The mean of η is at $y = 0$. 2-dimensional velocity is described by $\mathbf{V_i}(x, y, z) = (u_i, v_i, 0)$. In physical reality the system is situated on a rotating solid body, namely Earth, and so Coriolis acceleration is considered. The system is considered to be incompressible. Ω_1 and Ω_2 have respective constant densities ρ_1 and ρ_2 and constant non-zero vorticities γ_1 and γ_2.

A velocity potential φ_i and a stream function ψ_i are defined via $u_i = \partial_x\varphi_i - \gamma_i y = -\partial_y\psi_i$ and $v_i = \partial_y\varphi_i = \partial_x\psi_i$.

The following Bernoulli condition on the internal wave is established [2]

$$\rho_1((\partial_t\varphi_1)_c + \frac{1}{2}(\nabla\psi_1)_c^2 - (\gamma_1 + 2\omega)\chi_1 + g\eta) = \rho_2((\partial_t\varphi_2)_c + \frac{1}{2}(\nabla\psi_2)_c^2 - (\gamma_2 + 2\omega)\chi_2 + g\eta) \tag{5}$$

where $\chi_i := (\psi_i)_c$, g is the acceleration due to gravity and ω is the rotational speed of Earth. The kinematic boundary conditions are

$$\begin{cases} \partial_t\eta - \partial_x\eta\big(\gamma_i\eta - (\partial_x\varphi_i)_c\big) - (\partial_y\varphi_i)_c = 0 \text{ (on the internal wave)} \\ \mathbf{V_1}(x, -h_1, 0) = (u_1, 0, 0),\ \mathbf{V_2}(x, h_2, 0) = (u_2, 0, 0) \text{ (flatbed and lid respectively).} \end{cases} \tag{6}$$

The Hamiltonian is calculated in terms of conjugate variables $\eta(x,t)$ and $\xi(x,t)$ as

$$H(\eta,\xi)=\frac{1}{2}\int\limits_{\mathbb{R}}\xi\big(G_1(\eta)B^{-1}G_2(\eta)\big)\xi dx-\frac{1}{2}\int\limits_{\mathbb{R}}\rho_1\rho_2(\gamma_2-\gamma_1)^2\eta\partial_x\eta B^{-1}\eta\partial_x\eta dx$$
$$+\int\limits_{\mathbb{R}}\eta\partial_x\eta B^{-1}(\rho_1\gamma_1G_2(\eta)\xi+\rho_2\gamma_2G_1(\eta)\xi)dx+\frac{1}{6}\int\limits_{\mathbb{R}}(\rho_1\gamma_1^2-\rho_2\gamma_2^2)\eta^3dx+\frac{1}{2}\int\limits_{\mathbb{R}}(\rho_1-\rho_2)g\eta^2dx \tag{7}$$

where $\xi_i:=(\varphi_i)_c$ and $\xi:=\rho_1\xi_1-\rho_2\xi_2$. The Dirichlet-Neumann operator $G_i(\eta)$ and $B=\rho_1G_2(\eta)-\rho_2G_1(\eta)$ are also introduced.

The non-canonical equations of motion of the system are determined (cf. [3]) as

$$\begin{cases}\partial_t\eta=\delta_\xi H\\ \partial_t\xi=-\delta_\eta H+\Gamma\chi\end{cases} \tag{8}$$

which are non-canonical due to χ and the constant $\Gamma:=\rho_1\gamma_1-\rho_2\gamma_2+2\omega\big(\rho_1-\rho_2\big)$.

However, as ξ is defined modulo an additive constant it can be transformed, as per the single media case [5], to a new variable ζ giving canonical equations of motion via

$$\xi\to\zeta=\xi-\frac{\Gamma}{2}\int_{-\infty}^{x}\eta(x',t)dx'. \tag{9}$$

Finally, Hamiltonian structure is shown via (canonical) Poisson brackets in terms of η and ζ.

Acknowledgement. The author is funded by the DIT Fiosraigh Scholarship Programme. The author is grateful to R. Ivanov at Dublin Institute of Technology for valuable contributions.

References

[1] A. Compelli, Hamiltonian formulation of 2 bounded immiscible media with constant non-zero vorticities and a common interface. *Wave Motion* 54:115-124, 2015.

[2] A. Compelli, Hamiltonian approach to the modeling of internal geophysical waves with vorticity. *Monatsh. Math.* advance online publication, DOI:10.1007/s00605-014-0724-1, 2014; available from http://arrow.dit.ie/scschmatart/178/.

[3] A. Constantin, R. Ivanov and E. Prodanov, Nearly-Hamiltonian structure for water waves with constant vorticity. *J. Math. Fluid Mech.* 9:1-14, 2007.

[4] W. Craig, P. Guyenne and H. Kalisch, Hamiltonian long wave expansions for free surfaces and interfaces. *Comm. Pure Appl. Math.* 24:1587-1641, 2005.

[5] E. Wahlén, A Hamiltonian formulation of water waves with constant vorticity. *Lett. Math. Phys.* 79:303-315, 2007.

Mathematical Modelling of the Formation of Oscillation Marks in the Continuous Casting of Steel

Kevin Devine[†], **Prof. Stephen O'Brien**[†], **Dr. Michael Vynnycky**[‡], **Dr. Sarah Mitchell**[†], **Dr. Brendan Florio**[†]

[†]Mathematics Applications Consortium for Science and Industry (MACSI),
Department of Mathematics and Statistics
University of Limerick, Castletroy, Limerick, Ireland
kevin.devine@ul.ie, stephen.obrien@ul.ie, sarah.mitchell@ul.ie, brendan.florio@ul.ie

[‡]Division of Casting of Metals, Department of Materials Science and Engineering,
KTH Royal Institute of Technology, Brinellvägen 23, 100 44 Stockholm, Sweden
michaelv@kth.se

ABSTRACT

Continuous casting has been developed industrially worldwide since the 1950s as a high-throughput method for producing, amongst other things, metal billets, blooms and slabs; more than 90% of the world's steel is produced this way, amounting globally to more than one billion tonnes of steel cast per year. During casting, liquid steel is poured into the top of a water-cooled copper mould, where intense cooling causes a solidified steel shell to form. To prevent the steel sticking to the mould wall, a flux powder is added to the surface of the steel and the mould is oscillated at high frequency; the process is further complicated by low frequency phenomena associated with the turbulent flow of molten steel and the meniscus level fluctuations. All of these combine to produce undesired imperfections on the steel surface, which are expensive to remove, and a process that is both difficult to predict and control.

We revisit the work done by Hill et al. [2], which builds on earlier work [1], in order to formulate our model. This is a good starting point as it is more amenable than some numerical models [3, 4, 5, 6]. We use a lubrication approximation in the liquid flux region. Heat flow in the steel and flux is considered and coupled with the flow equations to predict mark formation. The model is non-dimensionalised in a systematic way, neglecting many of the assumptions made in [2]. We obtain what we believe to be the correct asymptotic model. We defer the numerical analysis arising from this model. We then proceed with our analysis by relaxing some dimensionless parameters. Thus a qualitative result is obtained.

Acknowledgement.
I would like to acknowledge the metal casting division in KTH, Stockholm, Sweden where I visited to complete this work. I would also like to thank the Irish Research Council who are funding my work.

References

[1] J. M. Hill and Y. H. Wu. On a nonlinear stefan problem arising in the continuous casting of steel. *Acta Mechanica*, 107(1-4):183–198, 1994. Cited By :8 Export Date: 27 June 2015.

[2] J. M. Hill, Y. H. Wu, and B. Wiwatanapataphee. Analysis of flux flow and the formation of oscillation marks in the continuous caster. *Journal of Engineering Mathematics*, 36(4):311–326, 1999. Cited By :14 Export Date: 27 June 2015.

[3] A. S. M. Jonayat and B. G. Thomas. Transient thermo-fluid model of meniscus behavior and slag consumption in steel continuous casting. *Metallurgical and Materials Transactions B: Process Metallurgy and Materials Processing Science*, 45(5):1842–1864, 2014. Export Date: 27 June 2015.

[4] P. E. Ramirez-Lopez, P. D. Lee, and K. C. Mills. Explicit modelling of slag infiltration and shell formation during mould oscillation in continuous casting. *ISIJ International*, 50(3):425–434, 2010. Cited By :18 Export Date: 27 June 2015.

[5] P. E. Ramirez-Lopez, P. D. Lee, K. C. Mills, and B. Santillana. A new approach for modelling slag infiltration and solidification in a continuous casting mould. *ISIJ International*, 50(12):1797–1804, 2010. Cited By :14 Export Date: 27 June 2015.

[6] P. E. Ramirez Lopez, K. C. Mills, P. D. Lee, and B. Santillana. A unified mechanism for the formation of oscillation marks. *Metallurgical and Materials Transactions B: Process Metallurgy and Materials Processing Science*, 43(1):109–122, 2012. Cited By :15 Export Date: 27 June 2015.

Remodeling in fibre-reinforced multiphasic biological Materials

Melania Carfagna

Politecnico di Torino
melania.carfagna@polito.it

Soft biological tissues are often quite heterogeneous. In most cases, they can be considered as deformable, fibre-reinforced porous media. This is for example the case of arterial walls and articular cartilage. Articular cartilage is usually modelled as a biphasic mixture, which consists of a deformable matrix, reinforced by collagen fibres, and an interstitial fluid flowing through and escaping from the tissue. The pathways followed by the interstitial fluid are influenced by the inhomogeneous behaviour of the solid matrix, and the preferential directions imposed by the local alignment of the fibres. The latter are statistically oriented in the tissue to optimize the mechanical loads. Indeed, three layers with distinct orientation of fibres can be identified experimentally in articular cartilage. Fibres are aligned vertically where the tissue touches the bone, they are randomly distributed in the middle, and parallel in the upper boundary. Thus, a preferential angle of orientation can be defined for each of these three layers. If the material undergoes large and/or permanent distortions, a reorientation and a structural reorganization of the fibres may occur, thereby leading to structural changes. Hence, dealing with remodeling seems to be of crucial importance to describe the transient response of the tissue to both physiological and pathological stimuli. Here we propose Finite Element simulations of unconfined compression of a sample of articular cartilage. The mechanical model accounts for the presence of anisotropy due to presence of fibres, which are numerically treated by means of a Spherical Design algorithm [1,2]. Moreover, the possibility of a transient reorganization of the fibres is addressed by solving an evolution law for the most probable angle of orientation of the fibres [3]. Some peculiar outcomes of the considered benchmarks are exposed and discussed.

References:

[1] Federico, S., Gasser, T.C., Nonlinear elasticity of biological tissues with statistical fibre orientation. J. R. Soc. Interface, 7, 955966 (2010).

[2] Tomic, A., Grillo, A., Federico, S., Poroelastic materials reinforced by statistically oriented fibresnumerical implementation and application to articular cartilage. IMA Journal of Applied Mathematics, 79(5), page 10271059 (2014).

[3] Grillo, A., Wittum, G., Tomic, A., Federico, S., Remodelling in statistically oriented fibre-reinforced materials and biological tissues. Mathematics and Mechanics of Solids, DOI: 10.1177/1081286513515265 (2014).

Gent models for the inflation of spherical balloons

Robert Mangan

School of Mathematics, Statistics and Applied Mathematics, NUI Galway
r.mangan4@nuigalway.ie

We revisit an iconic deformation of nonlinear elasticity: the inflation of a rubber spherical thin shell. We find the link between the exact solution of nonlinear elasticity and the membrane and Young-Laplace theories often used a priori in the literature. In particular, by expanding to first order in the thickness parameter, we recover the classical relation T =Pr/2, where T is the wall tension, P is the internal pressure and r is the radius of the sphere. We use the 3-parameter Mooney and Gent-Gent phenomenological models to explain the stretch-strain curve of a typical inflation, as these two models cover a wide spectrum of known models for rubber, including the Varga, Mooney-Rivlin, one-term Ogden, Gent-Thomas and Gent models. We find that the basic physics of inflation exclude the Varga, one-term Ogden and Gent-Thomas models. We compare the performance of both models on fitting the data for experiments on rubber balloons and animal bladder. We conclude that the Gent-Gent model is the most accurate and versatile model on offer for the modelling of rubber.

Waves in an infinite soft electroactive hollow cylinder under uniform biasing fields

Yipin Su

Department of Engineering Mechanics, Zhejiang University, Hangzhou 310027, China
su_yp@zju.edu.cn

Based on Dorfmann and Ogdens nonlinear theory of electroelasticity and the associated linear incremental theory, the non-axisymmetric wave propagation in an infinite incompressible soft electroactive hollow cylinder under biasing fields is investigated. The biasing fields are uniform, including an axial pre-stretch and a radial stretch in the plane perpendicular to the axis of the cylinder as well as an axial electric displacement. Such biasing fields make the originally isotropic electroactive material behave during its incremental motion like a conventional transversely isotropic piezoelectric material, hence greatly facilitating the following analysis. The three-dimensional equations of wave motion in cylindrical coordinates are derived and exactly solved by introducing three displacement functions. The exact solution is expressed in terms of Bessel functions, and explicit frequency equations are presented in different cases. For a prototype nonlinear model of electroactive material, numerical results are given and discussed. It is found that the initial biasing fields as well as the geometrical parameters of the hollow cylinder have significant influences on the wave propagation characteristics.

Participants

Faiza Alssaedi	*NUI Galway*	f.alssaedi1@nuigalway.ie
Valentina Balbi	*NUI Galway*	v.balbiv@gmail.com
Richard Burke	*NUI Galway*	richardburke8@gmail.com
Shane Burns	*NUI Galway*	s.burns3@nuigalway.ie
Melania Carfagna	*Politecnico di Torino, Turin*	melania.carfagna@polito.it
Alan Compelli	*Dublin Institute of Technology*	alan.compelli@mydit.ie
Martina Curran	*NUI Galway*	m.curran8@nuigalway.ie
Ioannis Dassios	*University of Limerick*	ioannis.dassios@ul.ie
Michel Destrade	*NUI Galway*	michel.destrade@nuigalway.ie
Kevin Devine	*University of Limerick*	kevin.devine@ul.ie
Patrick Fleming	*NUI Galway*	p.fleming3@nuigalway.ie
Roberto Galizia	*UNINA, Naples*	roberto.galizia@gmail.com
James Gleeson	*University of Limerick*	james.gleeson@ul.ie
Paul Greaney	*NUI Galway*	paul.greaney@nuigalway.ie
Naoise Holohan	*Trinity College Dublin*	nholohan@tcd.ie
Eilis Kelly	*Dublin Institute of Technology*	eilis.kelly@dit.ie
Dana Mackey	*Dublin Institute of Technology*	dana.mackey@dit.ie
Niall Madden	*NUI Galway*	niall.madden@nuigalway.ie
Robert Mangan	*NUI Galway*	r.mangan4@nuigalway.ie
Christine Marshall	*NUI Galway*	c.marshall1@nuigalway.ie
Niall McInerney	*University of Limerick*	niall.mcinerney@ul.ie
Martin Meere	*NUI Galway*	martin.meere@nuigalway.ie
Brendan Murray	*University College Dublin*	brendan.murray@ucdconnect.ie
Gary O'Keeffe	*University of Limerick*	gary.okeeffe@ul.ie
Paul O'Keeffe	*University College Cork*	115223612@umail.ucc.ie
Petri Piiroinen	*NUI Galway*	petri.piiroinen@nuigalway.ie
Brian Regan	*NUI Galway*	b.regan5@nuigalway.ie
Stephen Russell	*NUI Galway*	s.russell1@nuigalway.ie
Bram Siebert	*NUI Galway*	b.siebert1@nuigalway.ie
Eoghan Staunton	*NUI Galway*	e.staunton2@nuigalway.ie
Yipin Su	*Zhejiang University, China*	su_yp@zju.edu.cn
Szymon Urbas	*NUI Galway*	s.urbas1@nuigalway.ie
Keith Ward	*NUI Galway*	k.ward8@nuigalway.ie
Michael Welby	*NUI Galway*	m.welby5@nuigalway.ie
Giuseppe Zurlo	*NUI Galway*	giuseppe.zurlo@nuigalway.ie