DIFFERENCE SETS WITH CLASSICAL PARAMETERS IN ABELIAN GROUPS

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Let G be a group of order v. Let $D = \{d_1, d_2, \ldots, d_k\}$ be a k-subset of G. Then D is a (v, k, λ)-difference set for G if each non-identity $g \in G$ can be expressed in exactly λ ways as a product $g = d_i d_j^{-1}$ where d_i and d_j are elements of D.

If G is abelian and additive notation is used, the condition can be read that each non-zero $g \in G$ has exactly λ expressions as a *difference* $g = a - b$ where $a, b \in D$. Hence the origin of the term 'difference set'. For example, the set $\{1, 2, 4, 10\}$ is a difference set in $\mathbb{Z}_{13}$ as can be easily verified by computing the differences modulo 13.

Difference sets were introduced by James Singer in 1938. His family of difference sets have parameters

$$(v, k, \lambda) = \left(\frac{q^d - 1}{q - 1}, \frac{q^{d-1} - 1}{q - 1}, \frac{q^{d-2} - 1}{q - 1} \right)$$

where $d \geq 2$ and q is a prime power. Any diffference sets with these parameters are said to have classical parameters and there are several known families. These difference sets correspond to sequences with ideal autocorrelation properties and are of interest to applied mathematicians. There are several enticing but extremely difficult conjectures on these difference sets. Firstly, it is conjectured that q must be a prime power. Secondly, if such a difference set is in an abelian group, it is conjectured that the group must be cyclic. Such questions will not be addressed in this talk.

In this talk we will present some results on difference sets with classical parameters in abelian groups. In particular, we have studied how a difference set in a group interacts with subgroups and consequently how the parameters of a difference set can restrict the structure of the underlying group.

Our main result is that a planar difference set (giving rise to a finite projective plane) is embedded in a natural but not obvious way inside a difference set with the parameters of a 5-dimensional projective geometry.

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