

# On Sylow normalizers of finite groups

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We are interested in the influence of the Sylow normalizers, i.e., normalizers of Sylow subgroups, on the structure of finite groups. In particular, for some classes of groups, which can be seen as extension of nilpotent groups, we prove that a group belongs to the class if and only if its Sylow normalizers belong to the class. The classes appearing in our work are classes of groups with nilpotent Hall subgroups for well-defined sets of primes; as a particular case for instance, classes of groups which are direct product of Hall subgroups corresponding to pairwise disjoint sets of primes.

These results rest on the Sylow graph  $\Gamma_A(G)$  of a finite group  $G$ , with set of vertices  $\pi(G)$ , the set of prime divisors of the order of the group  $G$ , defined as follows: Two vertices  $p, q \in \pi(G)$  form an edge of  $\Gamma_A(G)$  if either  $q \in \pi(N_G(G_p)/G_p C_G(G_p))$  or  $p \in \pi(N_G(G_q)/G_q C_G(G_q))$ , where  $G_p$  denotes a Sylow  $p$ -subgroup of  $G$  for any  $p \in \pi(G)$ . We have proved that for a finite almost simple group  $G$ , the graph  $\Gamma_A(G)$  is connected and has diameter at most 5.

Our results generalize previous results of Glaubermann (1970), Bianchi, Gillio and Hauck (1986) and D'Aniello, De Vivo, Giordano and Pérez-Ramos (2005), related to the classes of finite  $p$ -groups, for a prime  $p$ , finite nilpotent groups and finite soluble groups with nilpotent Hall subgroups for prefixed sets of primes, respectively.