Groups in Galway 2015

1 Schedule

All talks will be in the Arts Millenium Building (AM200).

Friday 23 May

10.00–10.45	Nicolas Bergeron*
10.45–11.15	Coffee/tea
11.15–12.00	Tom De Medts
12.15–1.00	Götz Pfeiffer
1.00–3.00	Lunch
3.00–3.45	Hans Cuypers
4.00–4.45	Alain Valette
5.00–7:00	Poster Competition and Reception with refreshments**
7:30–	Conference Workshop Dinner***

* joint talk with VOAs+MMFs
** in the foyer of the Arts Millenium Building
*** The Ardilaun Hotel

Saturday 24 May

9.15–10.00	Radu Stancu
10.15–11.00	Serge Bouc
11.00–11.30	Coffee/tea
11.30–12.15	Peter Kropholler

2 Talk titles and abstracts

(1) **Nicolas Bergeron**. *Torsion homology and Bianchi modular forms*

Abstract: Arithmetic groups — that generalize the modular group — can have 'a lot' of torsion in their homology. Starting from the examples of congruence subgroups of the modular and Bianchi groups where homology reduces to abelianization, I will explain what 'a lot' means. I will then try to explain how this interacts with more classical question of number

theory and geometry. This is joint work with Akshay Venkatesh and Mehmet Haluk Sengun.

(2) **Tom De Medts**. *Jordan algebras and 3-transposition groups*

Abstract: A 3-transposition group is a finite group G generated by a conjugacy class D of involutions, such that the product of any 2 elements of D has order at most 3. These groups have been studied by Fischer in the 1970s, who discovered 3 new sporadic groups in this process.

Inspired by the theory of vertex operator algebras, Matsuo associated a commutative non-associative algebra to each 3-transposition group. Such an algebra is spanned by idempotents, and the eigenspaces for each of these idempotents satisfy very specific fusion rules, which happen to be the same as the (well-known) fusion rules for the Peirce decomposition in Jordan algebras.

This led us to the question whether some of these Matsuo algebras are also Jordan algebras. We give a complete answer to this question; this led to some rather unexpected results. Along the way, we also obtain results about Jordan algebras associated to (arbitrary) root systems.

This is joint work with Felix Rehren.

(3) **Götz Pfeiffer**. *On the Complexity of Multiplication in the Hecke Algebra*

Abstract: The Iwahori-Hecke algebra H of a finite Coxeter group W is an algebra with a basis in bijection to the elements of the group W, and a multiplication that is determined by deforming the multiplication in W. For the case where W is a symmetric group, I will discuss the complexity of multiplication in the Iwahori-Hecke algebra H when the elements of H are represented by simple coefficient lists, and how this complexity can be drastically improved when the combinatorics of the symmetric group is used to represent elements of H as nested lists. This is joint work with Alice Niemeyer and Cheryl Praeger.

(4) **Hans Cuypers**. *A Geometric Approach to classical Lie Algebras*

Abstract: An element x in a Lie algebra $\mathfrak{g}$ over a field $\mathbb{F}$ is called *extremal* if

$$[x, [x, \mathfrak{g}]] = \mathbb{F}x$$

(and satisfies some additional requirements when $\mathrm{char}(\mathbb{F}) = 2$.) Examples of extremal elements are the long root elements in classical Lie algebras.

Extremal elements are the focal point of the effort spearheaded by Cohen, *et al.*, to provide a geometric characterisation of the classical Lie algebras.

One can define a point-line space on the set of nonzero extremal elements of $\mathfrak{g}$ called the *extremal geometry* and denoted by $\mathcal{E}(\mathfrak{g})$. By combining the results in [1], [2] and [3] and of in 't panhuis and Cuypers, one can conclude that if $\mathfrak{g}$ is finite-dimensional, simple and generated by extremal elements, then $\mathcal{E}(\mathfrak{g})$ is the root shadow space of a spherical building. An interesting and important question is whether one can recover the algebra $\mathfrak{g}$ from the geometry $\mathcal{E}(\mathfrak{g})$.

Recently, Roberts, Shpectorov and Cuypers have answered this question affirmatively for buildings of type A, D and E, and, subsequently, Fleischmann and Cuypers for all spherical buildings of type different from G_2.

By using the theory of buildings first introduced by Tits in [4], this provides a geometric interpretation of the classical Lie algebras analogous to his own achievement for the algebraic groups.

References

[1] A. M. Cohen and G. Ivanyos, Root filtration spaces from Lie algebras and abstract root groups, *Journal of Algebra* **300** (2006), 433–454.

[2] A. M. Cohen and G. Ivanyos, Root Shadow Spaces, *European Journal of Combinatorics* **28** (2007), 1419–1441.

[3] A. Kasikova and E. Shult, Point-line characterizations of Lie geometries, *Adv. Geometry* **2** (2002), 147–188.

[4] J. Tits, *Buildings of spherical type and finite BN-pairs*, (Springer, 1974).

(5) **Alain Valette**. *Box spaces: bridging geometric and asymptotic group theory*

Abstract: If G is a finitely generated, residually finite group and (N_i) is a sequence of finite index normal subgroups decreasing to the identity, the associated box spaces is the disjoint union of the G/N_i's, with a suitable metric. Any box space of a property (T) group is an expander, by a result of Margulis (1973). An important question in geometric group theory is:

Which group properties are captured by a box space, up to coarse equivalence?

The talk will survey the known results, ending with work in progress with Ana Khukhro.

(6) **Radu Stancu**. *Evaluations of simple biset functors*

Abstract: Let G be a finite group and k be a field. The double Burnside module $kB(G,H)$ is the Grothendieck group of isomorphism classes of (G,H)-bisets. When $G = H$, the module $kB(G,G)$ has an internal multiplication, making it into a ring : the double Burnside ring.

It turns out that the simple $kB(G,G)$-modules are evaluations at G of simple biset functors. The evaluation of simple functors can be zero, giving rise to the phenomenon of vanishing evaluations of functors, which is very hard to detect in general. As recently remarked by Rognerud, the control of this phenomenon gives information on the global dimension of $kB(G,G)$.

The purpose of this talk is to give criteria to decide whether an evaluation of a biset functor vanishes. Under some conditions on the poset of sections of G, isomorphic to a given subquotient H, a closed formula for the evaluation of the simple functor $S_{H,V}$ is given.

The talk presents a joint work with Serge Bouc and Jacques Thévenaz.

(7) **Serge Bouc**. *Representations of finite sets*

Abstract: In this joint work with Jacques Thévenaz, we develop the representation theory of finite sets and correspondences: for a commutative ring k, we consider correspondence functors, i.e. k-linear functors from the category of finite sets, where morphisms are k-linear combinations of correspondences, to the category of k-modules. We discuss various properties the abelian category $\mathcal{F}_k$ of correspondence functors: finite generation and finite length, self-injectivity and symmetry, global dimension, projective, injective, and simple objects, tensor structure, etc. We also associate a correspondence functor F_T to any finite lattice T, and show that F_T is projective if and only if T is distributive. This construction extends to a fully faithful k-linear functor from a suitable category $k\mathcal{L}$ of finite lattices to $\mathcal{F}_k$.

(8) **Peter Kropholler**. *New lines of attack on soluble groups and cohomology*

Abstract: We'll survey some recent work of various authors on cohomological finiteness conditions for soluble groups and closely related groups such as RAAGs and amenable groups.

The survey will include a discussion of my own joint work with Karl Lorensen, recent work of Ian Leary and of Ged Corob Cook.

Here is Leary's very beautiful recent result: there are uncountably many groups of type FP_2.

And here is Corob Cook's wonderful result: soluble pro-p groups of type FP_∞ are polycyclic.

And if you want to know what Karl and I are up to, come to the lecture.