

Nonparametric Predictive Utility Inference

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Abstract

This work considers the combination of two strands of recent statistical research: that of decision making with uncertain utility and that of nonparametric predictive inference. In doing so we discuss the use of Nonparametric Predictive Utility Inference (NPUI) within a sequential decision selection problem for the situation of a Decision Maker (DM) who is confronted with a choice set that includes novel or unfamiliar outcomes.

Traditionally subjective expected utility theory has been suggested for reconstructing the DM's preferences over a collection of decisions by taking into account both the probability that each decision leads to a particular outcome, and the DM's relative preference for obtaining that outcome as measured by its utility value. However, implicit within this theory (and hence necessary for its application), is the assumption that the DM knows their preferences, meaning they can assign an appropriate utility function for use within the problem which has domain the full set of all possible decision outcomes.

Nevertheless, not for all situations is it appropriate to assume a known preference relation over outcomes. Indeed, it may be necessary for a DM to experience a novel outcome before being able to assign an appropriate utility value for its selection. Such situations of utility uncertainty motivate Cyert & DeGroot's adaptive utility theory [1], where a utility function is only assumed known up to the value of some uncertain parameter. Houlding further extended these ideas and defined an adaptive utility function to be the expected value of the supposed true utility function with respect to prior beliefs over the uncertain utility parameter. Yet, despite the theory explicitly permitting a DM to remain uncommitted to a presumed known and correct utility function, previous use has required knowledge of a precise prior distribution concerning preferences, a requirement that is unlikely to be reasonable in practice.

As an alternative to explicitly declaring a precise prior distribution over an uncertain utility parameter, interest here lies in the use of Nonparametric Predictive Inference (NPI), which is a low structure statistical technique arising naturally as a result of Hill's $A_{(n)}$ assumption [2, 3]. Given an ordered series of observations that are considered subject to a pre-data exchangeability assumption, NPI proceeds by assigning equal mass to the probability that a new observation falls within any of the intervals formed by the known ordered data, leading to the quantification of uncertainty through interval probability.

This paper discusses the use of NPUI within the setting of a sequential decision problem and considers the implications of various decision selection criteria that may be considered within the framework. It is argued that NPUI is a generalisation of traditional Bayesian decision theory, but one that is less demanding of the DM by allowing her to learn her preferences through experience as she proceeds.

References

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