Stokes Modelling Workshop:

National University of Ireland, Galway.
June 23 - 26, 2014

Organising Committee:
Shane Burns, John Donohue, Artur Gower.

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Problem Descriptions.

Bouncing Balls

Requires the skills: Mechanics and differential equations

Consider the case of two balls being dropped together, for example the top ball being a baseball and the bottom ball a basketball. It has been observed experimentally that after impact the bottom ball will remain close to the ground whereas the top ball will bounce extremely high.

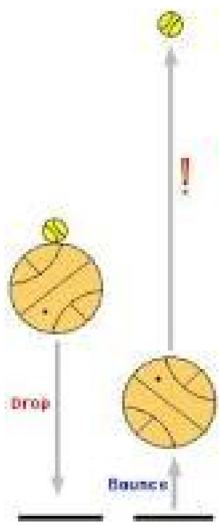

Figure 1: Basketball baseball experiment

Question: How can we model this mathematically?

Seek and Destroy

You are about to compete in the final game of the Robot Wars League. In each game of the league, two robots, each constructed by rival competitors, battle each other in a 30 m × 30 m square arena. Robots cannot be controlled by their competitors in real time and the robots are pre-programmed with a set of instructions that they must follow in the arena. Games are 5 minutes long and result in a draw if both robots are still operational at full-time. Each robot is fitted with a GPS tracker and is aware of their position within the arena and the position of their opponent at all times during the game. This information can be taken as input for the set of instructions the robot must follow.

Your robot is "Predator". Predator has a maximum speed at 5 m/s and has a turning radius of 2 m. You have been drawn against "Prey". Prey has a maximum speed of 4 m/s and has a turning radius of 1 m. In designing Predator, you have sacrificed agility for power and speed. Predator is so powerful that if he catches Prey, you are confident that he will destroy him. You must win this final game in order to win the league. A loss or a draw will result in Prey winning the league. Obviously, the best strategy for Prey is to try to force a draw. You must decide upon a programme for Predator to follow.

You can assume that the 2-D projections of Predator and Prey onto the floor form circles of diameter 1 m. You can also assume that both robots can take initial positions anywhere inside 1 m × 30 m rectangles at opposing ends of the arena, as shown in Figure 2.

In your final report, feel free to compare different strategies, if you wish. However, try to decide on one strategy that you plan to use and provide your reasoning behind this choice.

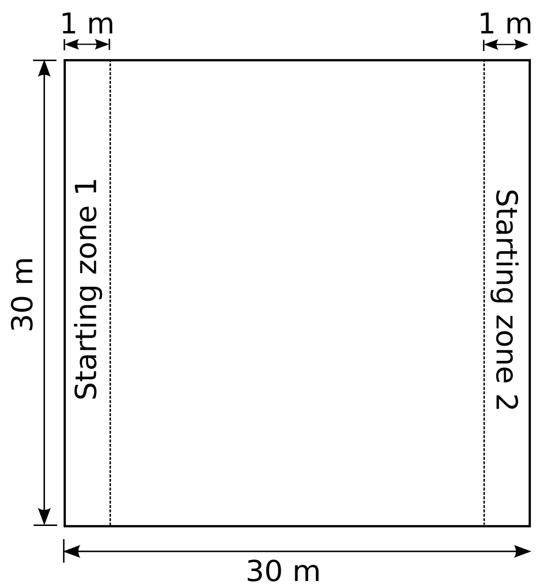

Figure 2: The arena dimensions, with the starting zones labelled.

Carving with a Water Jet

Requires the skills: Differential Equations and Mechanics.

There is a very new technique in industry for carving surfaces in stone and metal with a high powered jet of water with small pieces of stone. Here is a picture

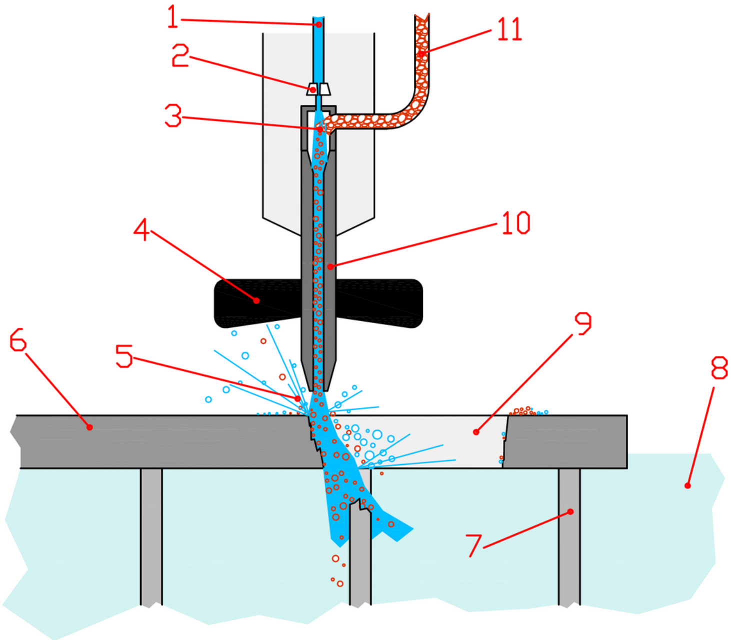

Figure 3: Illustration of the water jet carving a flat plate of steel. Ignore the numbers.

Note that the jet's nozzle has a reasonable thickness (not like a laser) which complicates the matter, and the nozzle is circular. To carve a specific surface engineers have been using trial and error, with little success, as there is yet no good mathematical model.

To simplify let us consider a water jet living in 2D. To help understand how the jet works several experiments can be devised. For example, Figure 4 shows the profile the jet carves out into a flat plate of steel when it is turned on for one second and pointed straight down at $x = 0$ with the nozzle at a height of $y = 1$.

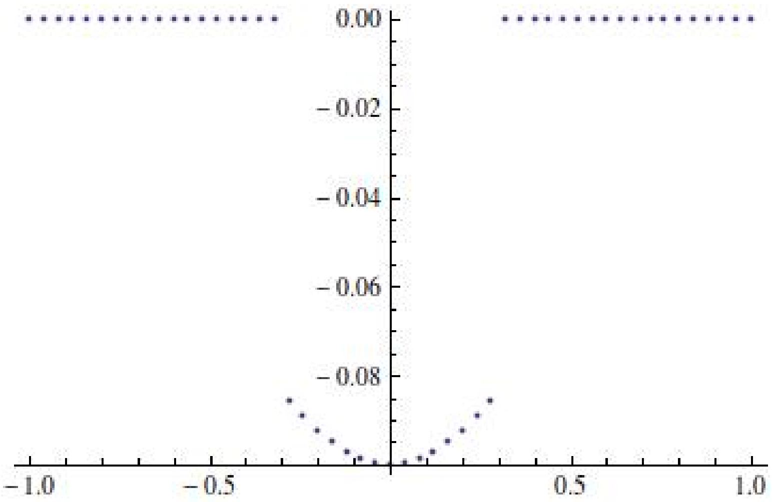

Figure 4: If the Jet is standing still and turned on for 1s this is the resulting profile it carves out into steel. The units used are centimeters.

Summary: given the jet's trajectory and velocity can you devise a model that determines the surface that will be carved? As an input to your model you can use experimental results such as the above, or any other reasonable experiment.

Extracting Manufactured Errors

Requires one of the skills: Linear Algebra, Statistics or Optimization.

A lot of analysis and mathematics goes into designing the optimal shape for helicopter blades'. This optimal shape is called the *design intent.* However when manufacturing the piece tiny errors are introduced, and in the extreme conditions of flight, tiny errors on a helicopter blade can be fatal. The same issue occurs when manufacturing many high performance pieces, specially those used in flight where tiny errors can cause turbulence.

To simplify this situation imagine the design intent is just one flat plate. In this case there are only 3 types of errors that can be introduced during manufacturing: miss aligned plates, waviness or bolts, see Figure 5 below. To manufacture the one flat plate several plates need to be bolted and stuck together. Sometimes two plates are ever so slightly misaligned causing a step or an inclination. The waviness is due to one of the plates buckling under compression, and the bolts can protrude too far or be in the wrong place from an aerodynamics perspective.

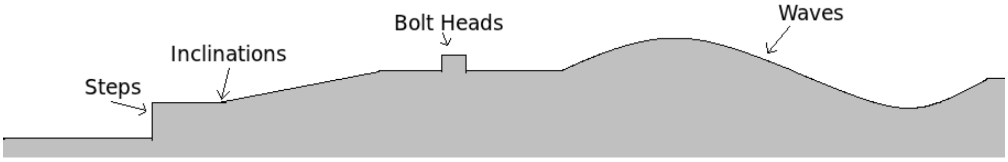

Figure 5: When attempting to manufacture a flat plate several errors can be introduced.

When the piece is manufactured it needs to be assessed if it will do the job it was made for. In this crucial phase the company needs a simple mathematical description of the manufacture piece that they can input into their simulations (another way of testing the piece would be to, for example, place the blades on a helicopter and see if it will fly. This approach can end badly). This description needs to identify what errors are present and their location, i.e. there is a 2mm step that starts at 1m along the piece.

To build this mathematical description the company will give you a measurement of the manufactured piece, this measurement will consist of thousands of points in (x, y). If the piece was manufactured correctly then $y = 0$ for all the points (remember we are trying to make a flat plate). So x is how far along the manufactured piece and y is the size of the error. Note that the high precision lasers used to make this measurement do not give evenly spaced points along x.

What makes this task challenging is that the lasers also introduce errors, in the form of white noise, which can be of the same size as the errors introduced by the manufacturing process! Here is an example:

Summary: given the measured data consisting of thousands of (x, y) points, can you create a method to approximate the manufactured piece (that is ignore the white noise introduced by the measurement) and at the same time identify if and where there are any steps, inclinations, bolts or waviness?

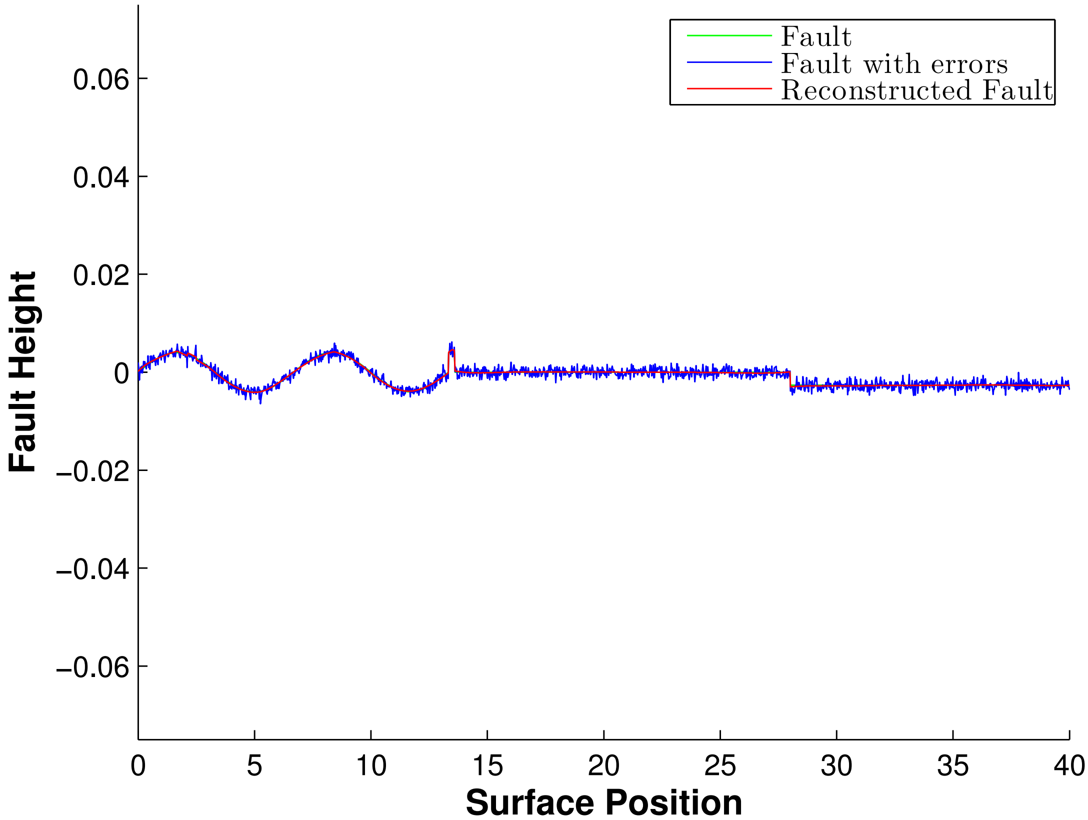

Figure 6: Blue is what was measured by the lasers, and the red ignores the lasers errors and attempts to identify what was really manufactured.