

PROJECT SUGGESTIONS 2015-2016 (J.B.)

1. The Complex Projective Plane, Complex Conjugation and the Four Dimensional Sphere:

There is a striking result which says that the quotient space of the complex projective plane by complex conjugation (of its homogeneous coordinates) is the 4-sphere. Having covered the prerequisite material, a proof of this result by W.S. Massey (Geom. Dedicata 1972) will be studied.

2. Grothendieck's Theorem on Algebraic Vector Bundles on the Projective Line:

The theorem referred to above states that an algebraic (or holomorphic) vector bundle on the projective line splits into a sum of line bundles the isomorphism classes of which are uniquely determined by the vector bundle. Each line bundles in turn is determined by an integer (its Chern number). Having covered the prerequisite material (surprisingly small), a proof of this result by M. Hazewinkel and C. Martin (J. Pure and Applied Algebra 1982) will be studied.

3. Minimal and Constant Mean Curvature Surfaces:

Minimal surfaces are those having least area among all surfaces with the same boundary. Having covered the prerequisite material on minimal and the close cousins, constant mean curvature the project will focus on reading a research paper on such surfaces having rich symmetry properties.

4. The 27 Lines on a cubic surface:

The celebrated Cayley–Salmon theorem says that all non-singular cubic surfaces (in complex projective 3-space) contain precisely 27 lines. Having covered the prerequisite material from Algebraic Geometry, a recent proof of the theorem by T. Tau based on classical differential geometry will be studied. Time permitting, a recent proof of the fact that at most 5 of these lines are twistor lines, will also be studied.

5. A Geometric approach to Lie Algebra Theory: (Postgrad project)

This project will be based on reading the recent paper “Lie Algebra Theory without Algebra” by S.K. Donaldson. The paper is concerned with geometric proofs of the standard results about maximal compact subgroups of simple Lie groups.