Train positioning and track location using video odometry and track curvature

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Problem and Objectives

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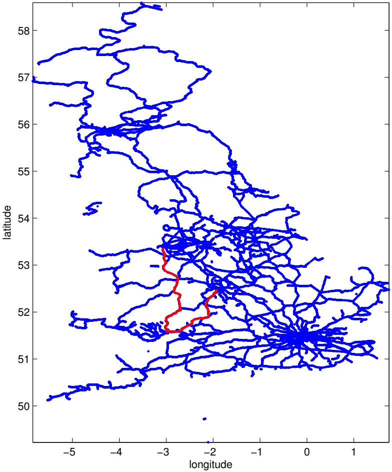
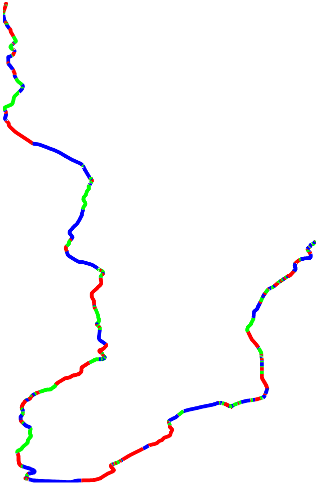
2. Carefully calculate (i) the train velocity (ii) the track curvature
3. Assimilate ALL the data to find the train location

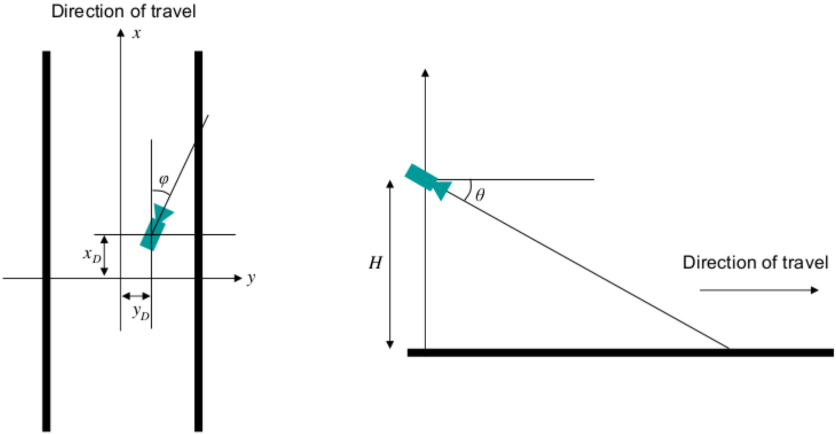
Camera Snapshot

Generate Rail Network





Turn Trains into Maths



Three angles for camera calibration: pitch θ, yaw ϕ, roll ψ.

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What does the camera see? Take a point $(X, Y, -H)$ relative to the camera position.

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$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = R_1(\psi)R_2(\theta)R_3(\phi)\begin{pmatrix} X \\ Y \\ -H \end{pmatrix},$$

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Squash point onto camera lens,

$$\begin{pmatrix} u \\ v \end{pmatrix} = f \begin{pmatrix} \eta/\xi \\ \zeta/\xi \end{pmatrix},$$

where f is the focal length of the camera.

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This poor shaking camera has to see a slight curvature in the rail

- Lateral curvature

$$Y = Y_0 + \beta \frac{X^2}{2},$$

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We can work with small $\delta\theta$, $\delta\phi$, $\delta\psi$, δH, δY, α, β, for it allows us to linearize...

The Signatures from $\delta\theta$, α, etc...

Each $\delta\theta$, α, etc... produces an independant *signature*,

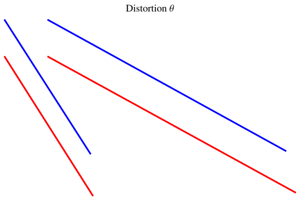

Figure: $\delta\theta$ distortion.

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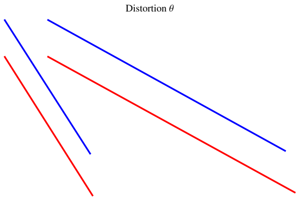

Figure: $\delta\theta$ distortion.

Red are the straight perfectly measured rail.
Blue are the linearised disturbance to the rail.

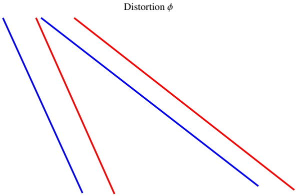

Figure: $\delta\phi$ distortion.

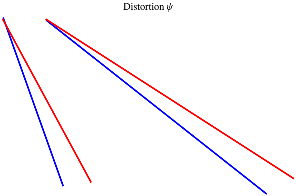

Figure: $\delta\psi$ distortion.

The Signatures from $\delta\theta$, α, etc...

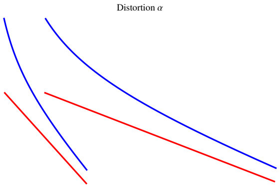

Figure: α distortion.

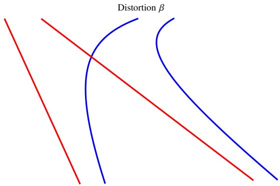

Figure: β distortion.

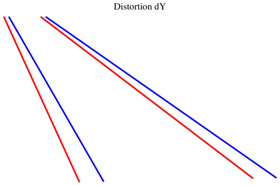

Figure: δY distortion.

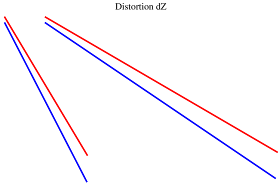

Figure: δH distortion.

Signature Projection

Each signatures becomes a vector

$$\mathbf{B} = \left(\mathbf{V}_{\delta\theta}|\mathbf{V}_{\delta\phi}|\mathbf{V}_{\delta\psi}|\mathbf{V}_{\alpha}|\mathbf{V}_{\beta}|\mathbf{V}_{\delta H}|\mathbf{V}_{\delta Y}\right)$$

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We can project the difference $\mathbf{V}$ from the observed rails and the straight perfectly measured rail,

$$\begin{pmatrix}\delta\theta & \delta\phi & \delta\psi & \alpha & \beta & \delta H & \delta Y\end{pmatrix}^T = \left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{V}$$

The determinant

$$\det\left(\mathbf{B}^T\mathbf{B}\right)$$

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The determinant

$$\det \left(\mathbf{B}^T \mathbf{B} \right) \leftarrow \text{ a function of } \theta,\ \phi,\ \psi,\ H \text{ and } dY.$$

can be used to optimize camera position.