# Counter-Inuitive Acoustoelasticity

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## Abstract

The determination of the direction of greatest tension in a deformed solid is one of the main goals of acoustic non-destructive evaluation because, for isotropic solids, this direction coincides with the direction of greatest stress.

In this poster, we present results of acoustic waves on deformed elastic solids, where the wave speed does not have its greatest value along the direction of greatest stretch [4]. This goes against what was commonly accepted.

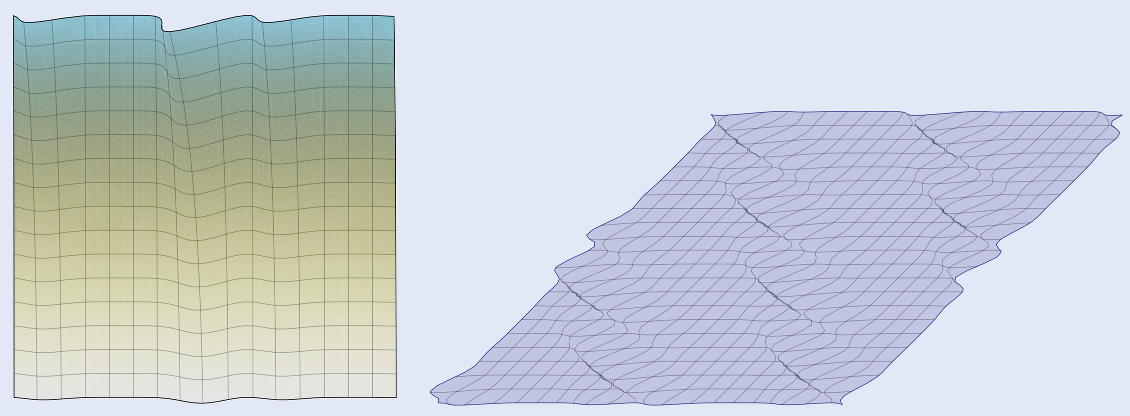
## Surface Acoustics

Let $\phi : X \mapsto x$ be a finite elastic deformation. To add a surface acoustic [2] we solve for a map

$$X \mapsto x + \mathbf{U}(x_2)\mathrm{e}^{\mathrm{i}k(x_1\cos\theta + x_3\sin\theta) - \omega t},$$

in the first order of $\|\mathbf{U}(x_2)\|$, with the restrictions of zero traction on $x_2 = 0$ and that the solutions decays $\lim_{y\to\infty} \mathbf{U}(y) = 0$. An example is given in "Graphics" with a shear as the finite deformation consider

## Graphics



a) b)

Figure **a)** is a slice of a surface wave along $x_2$, note how the wave's amplitude decays, and **b)** is a bird's eye view of the surface $x_2 = 0$.

## Conditional Impedance Method

Rather than solving for a particular displacement $\mathbf{U}(0)$ , with resulting normal stress $-\mathrm{i}\mathbf{V}(0)$, on the surface $x_2 = 0$, we solve for the conditional impedance $\mathbf{Z}(\omega,\theta)$, see [1], [4] and [3], where

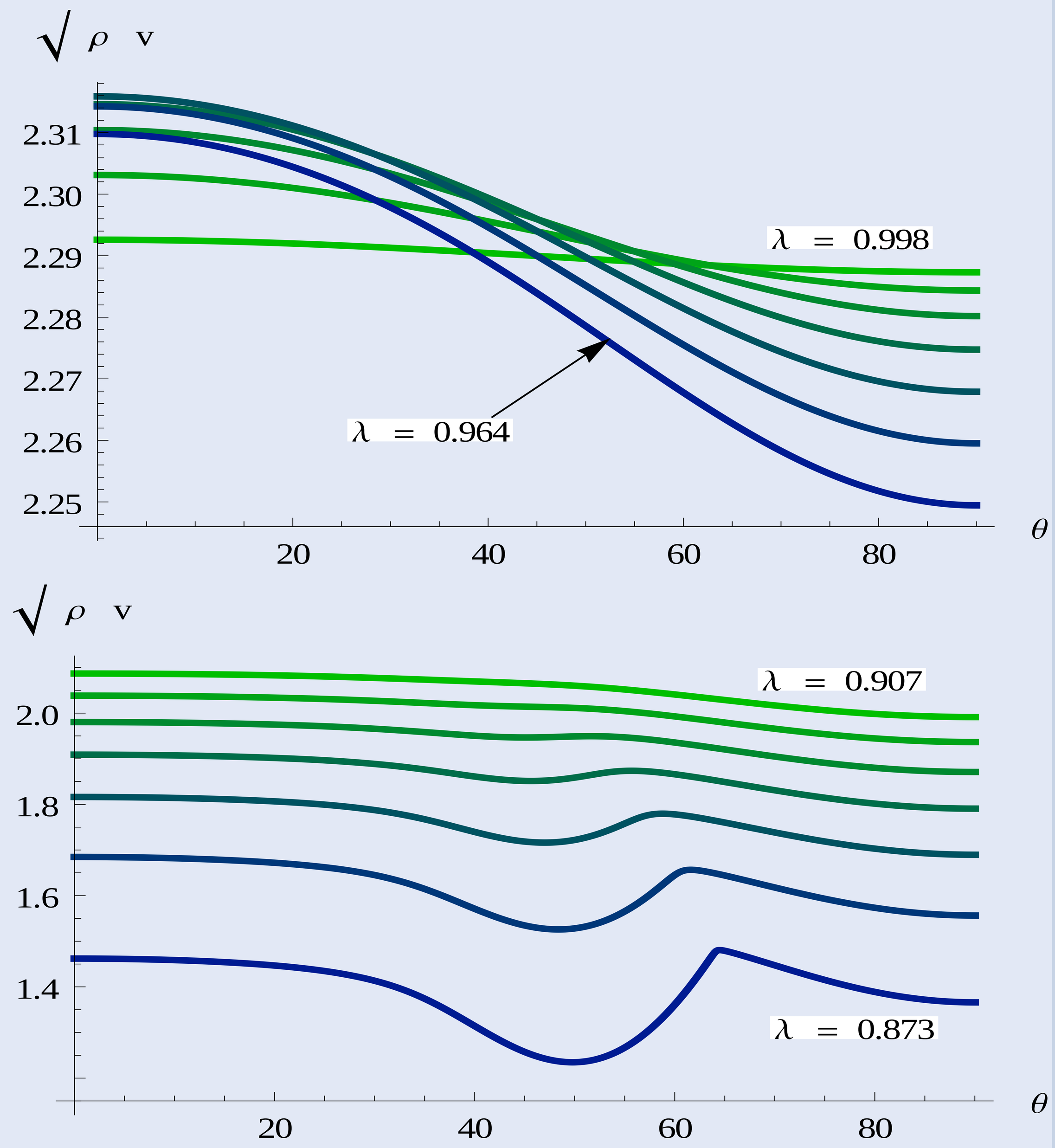
$$\mathbf{V}(0) = -\mathrm{i}\mathbf{Z}(\omega,\theta)\mathbf{U}(0), \quad \text{and} \quad (\mathbf{Z} - \mathrm{i}\mathbf{R}^{\mathrm{T}})\mathbf{T}^{-1}(\mathbf{Z} + \mathrm{i}\mathbf{R}) - \mathbf{Q} + \rho\omega^2/k^2\mathbf{I} = 0,$$

where $\mathbf{R}$, $\mathbf{T}$ and $\mathbf{Q}$ are given in terms of the fourth-order elasticity tensor, which in turn depends on $\theta$ and the underlying deformation ($\lambda$). The conditional impedance arises naturally [4] from

$$\underbrace{\mathbf{U}^*(0)\cdot k^2\mathbf{Z}(\omega,\theta)\mathbf{U}(0)}_{\textbf{Surface Stress Power}} = \underbrace{k\int_0^\infty \delta W(\mathbf{U}(y))dy}_{\textbf{Potential Energy}} - \underbrace{\omega^2\int_0^\infty \rho\langle\mathbf{U}^*(y),\mathbf{U}(y)\rangle dy}_{\textbf{Kinetic Energy}},$$

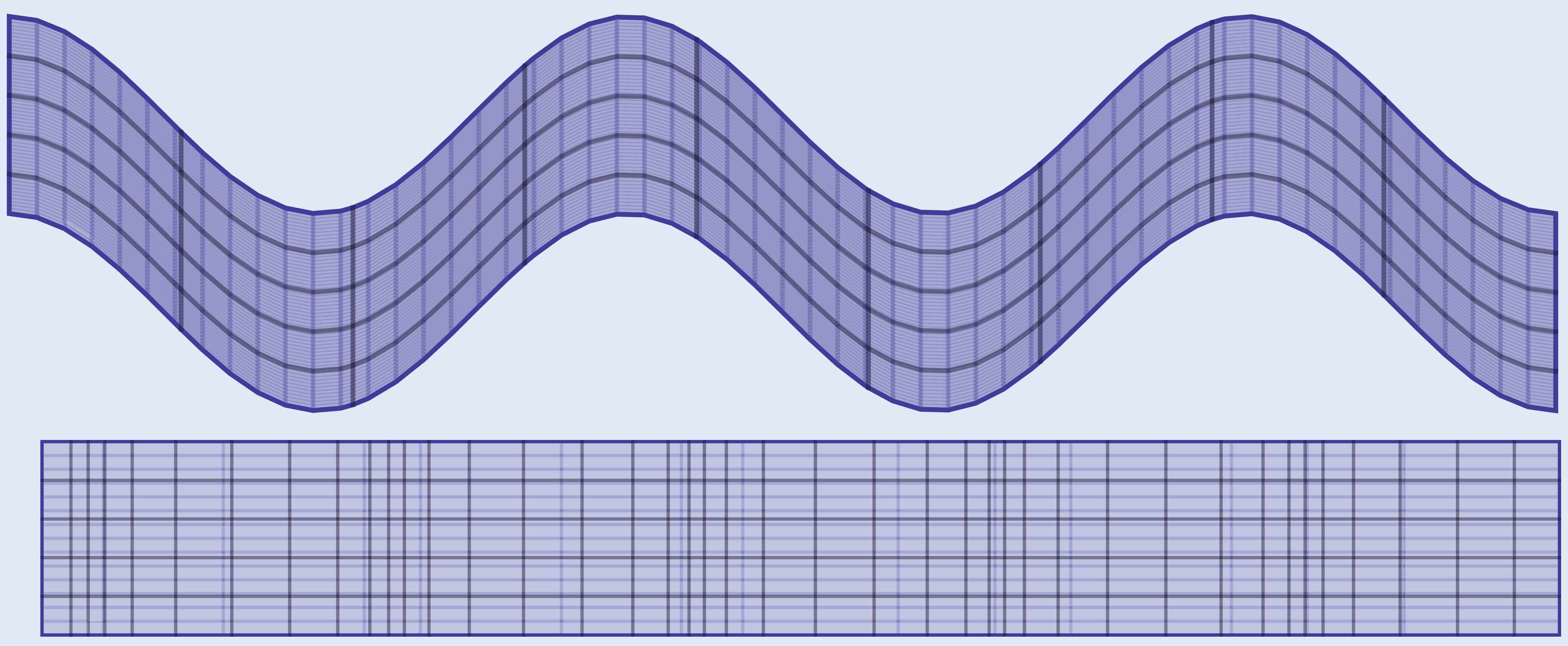
implying that $Z(0,\theta) > 0$ and $\dfrac{\partial Z}{\partial\omega}(\omega,\theta) < 0$, both superb for finding $\omega^*$ such that $\det Z(\omega^*,\theta) = 0$, which gives a stress free surface. These properties, together with $\mathbf{Z}$ being given by a well understood algebraic equation, allowed us to reliably calculate surface waves on materials with higher-order nonlinearity, which in turn lead to finding the phenomena in question.

## Results




The Figure shows the surface wave speed for Nickel under uniaxial isochoric compression. The coordinates $x_1$ and $x_3$ correspond to the directions of greatest compression and stretch. The compression ratio along $x_1$ is given by $\lambda$.

## Body Waves



a)

b)

Surface waves are usually composed of both transversal, figure **a)**, and longitudinal, figure **b)**, body waves. We can see that **a)** stretches the material in one direction, while in an orthogonal direction it alternatively compresses and stretches the material. The propagation speed of a wave is proportion to the (time-averaged) potential energy increase caused by that wave. So waves primarily of the form **a)** travel fastest along the most stretched directions. On the other hand, waves primarily of the form **b)** can, in a nonlinear deformed isotropic solid, cause a larger increase in potential energy along a compressed direction than a stretched direction.

## Conclusion

This phenomena was uncovered by using a new highly reliable method based on the conditional impedance matrix. The results can be thought to be counter-intuitive for they due to the well known results for neo-hookean materials and wave speeds in stretched violin string. However, on a closer inspection of the physics, in a nonlinear solid it is natural for longitudinal waves to travel fastest along a compressed direction. So a surface wave composed of both longitudinal and transversal waves, coupled with a boundary condition $\det \mathbf{Z} = 0$, can exhibt a wide variety of velocity profiles.

## References

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## Acknowledgements

This work is conducted under the supervision of Prof. M. Destrade and is funded by a Hardiman Scholarship from NUI Galway. I also thank the support from my collegues at the Riverside Terrapin.