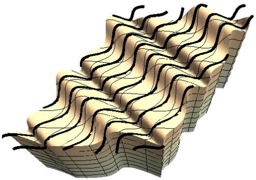

Characterizing Fibre Reinforcement through Surface Wrinkles

Author:
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National University of Ireland Galway

NUI Galway
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Fibre Reinforced Tissues (one preferred direction)

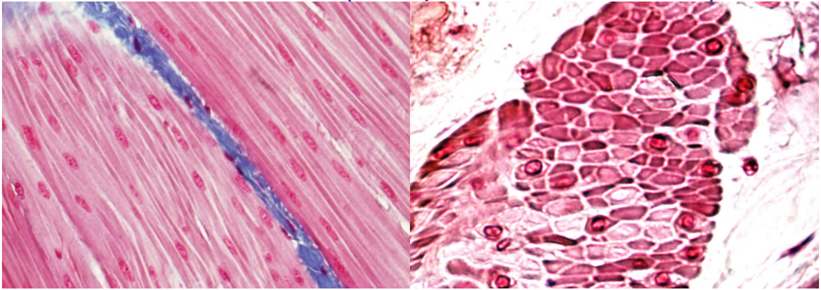

Figure: Smooth muscles cells cut along the cells and cross section.

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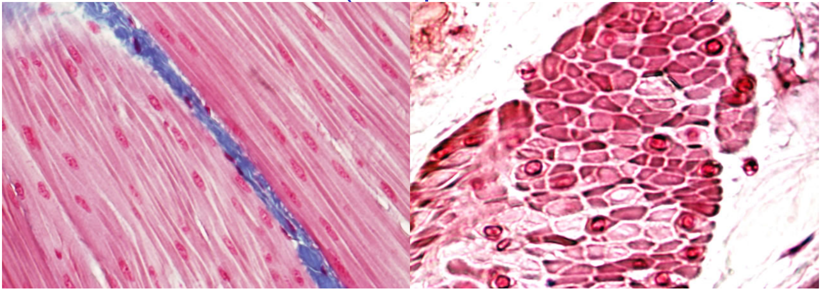

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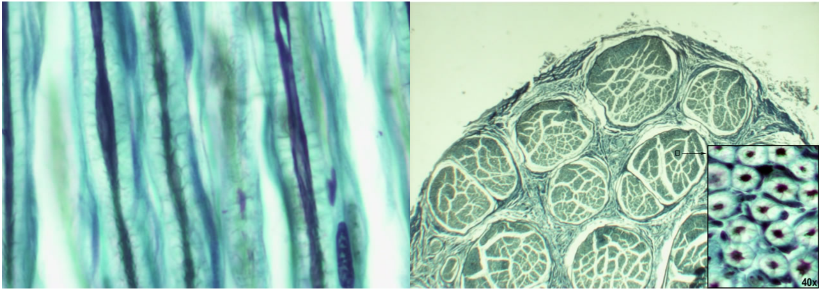
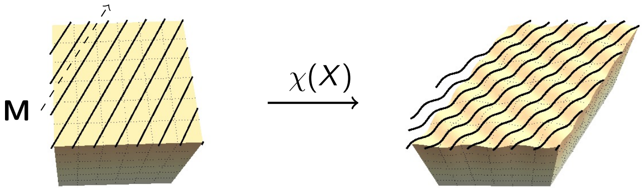


Figure: Sciatic nerve cells cut along the cells and cross section.

Phenomena on a larger scale.



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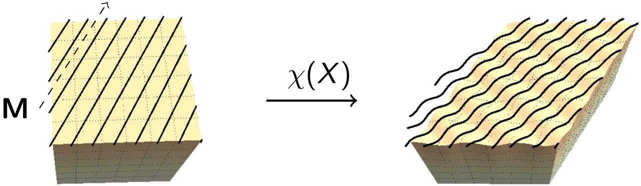

Potential energy $W = W(I_1, I_2, I_3, I_4, I_5)$

$$I_1 = \mathrm{tr}\mathbf{C}, \quad I_2 = (\mathrm{tr}\mathbf{C})^2/2 - \mathrm{tr}(\mathbf{C}^2)/2, \quad I_3 = \det\mathbf{C},$$
$$I_4 = \mathbf{M}^T\mathbf{C}\mathbf{M}, \quad I_5 = \mathbf{M}^T\mathbf{C}^2\mathbf{M},$$

where $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ and $\mathbf{F} = \boldsymbol{\nabla}\chi(X)$.

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So let's hang on to both I_4 and I_5.

Separating the Role of I_4 and I_5.

There are many choices for how to include the anisotropic invariants in W, for example [Holzapfel and Ogden, 2010, Lu and Zhang, 2005], [Ciarletta et al., 2011, Horgan and Saccomandi, 2005].

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- How about clearly separating the influence of both invariants?

- How about modelling fibre resistance to compression?

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$$\mathbf{C}^3 - \mathbf{C}^2 I_1 + \mathbf{C} I_2 - \mathbf{I} I_3 = \mathbf{0} \implies \mathbf{C}^2 - \mathbf{C} I_1 + \mathbf{I} I_2 - \mathbf{C}^{-1} I_3 = \mathbf{0},$$

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contracting $\mathbf{M}^T$ on the left and $\mathbf{M}$ on the right

$$I_5 = \mathbf{M}^T \mathbf{C}^2 \mathbf{M} = I_4 I_1 - I_2 + \mathbf{M}^T \mathbf{C}^{-1} \mathbf{M} I_3.$$

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So instead of I_4 and I_5 we can use

$$I_4^S = I_4 = \mathbf{M}^T \mathbf{C} \mathbf{M} \quad \text{and} \quad I_4^C = \mathbf{M}^T \mathbf{C}^{-1} \mathbf{M}.$$

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Clear interpretation, for example $W = W(I_4^S, I_4^C)$

$$\boldsymbol{\sigma} = 2\partial_{I_4^S} W \mathbf{m}^S \otimes \mathbf{m}^S - 2\partial_{I_4^C} W \mathbf{m}^C \otimes \mathbf{m}^C,$$

with $\mathbf{m}^S = \mathbf{F}\mathbf{M}$ and $\mathbf{m}^C = \mathbf{F}^{-T}\mathbf{M}$.

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Prototype:

$$W_A = \frac{A_S}{4}(I_4^S - 1)^2 + \frac{A_C}{4}(I_4^C - 1)^2.$$

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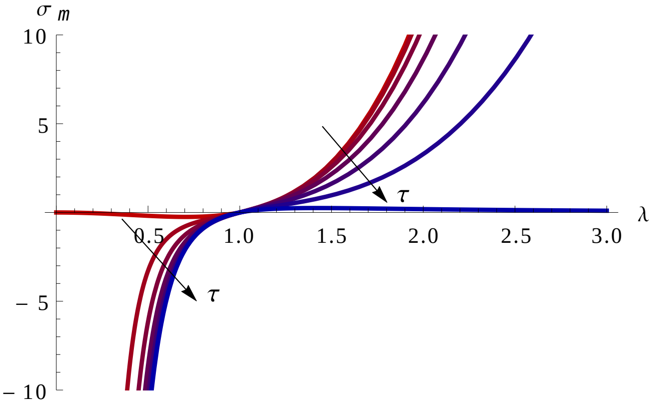

Figure: $\mathbf{M}^T \boldsymbol{\sigma} \mathbf{M} = \sigma_m$ with $A_S = \cos\tau$ and $A_C = \sin\tau$.

Surface Wrinkles as a Measurement

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Determining the model's parameters from experiments is a major challenge when using both anisotropic invariants.

- Surface wrinkles can assist in characterizing the material.
- Changing the contribution of I_4^S vs I_4^C should signifcantly alter the resulting wrinkles.

Surface Wrinkles

We look for $\delta\chi(x_1, x_2, x_3) = \mathbf{U}(x_2)\mathrm{e}^{\mathrm{i}k(x_2\cos\theta + x_3\sin\theta)}$, $\mathbf{x} = \chi(X)$

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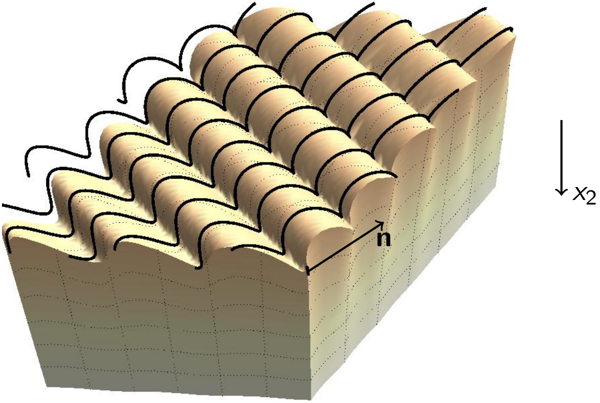

Figure: A surface-wrinkle with $\mathbf{n} = (\cos\theta, 0, \sin\theta)$.

The surface-wrinkle's amplitude decays as x_2 increases.

(Show gif of a wrinkle forming, more technical see [Gower, 2014])

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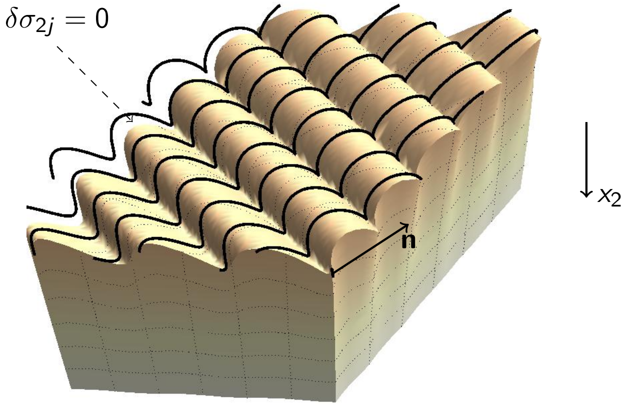

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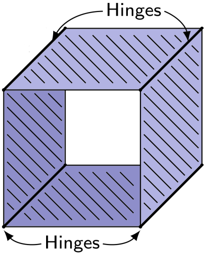
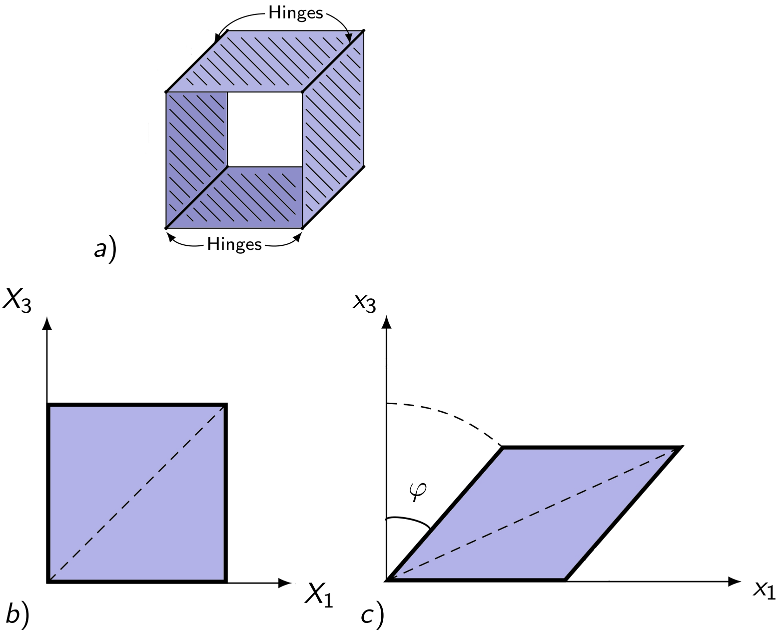
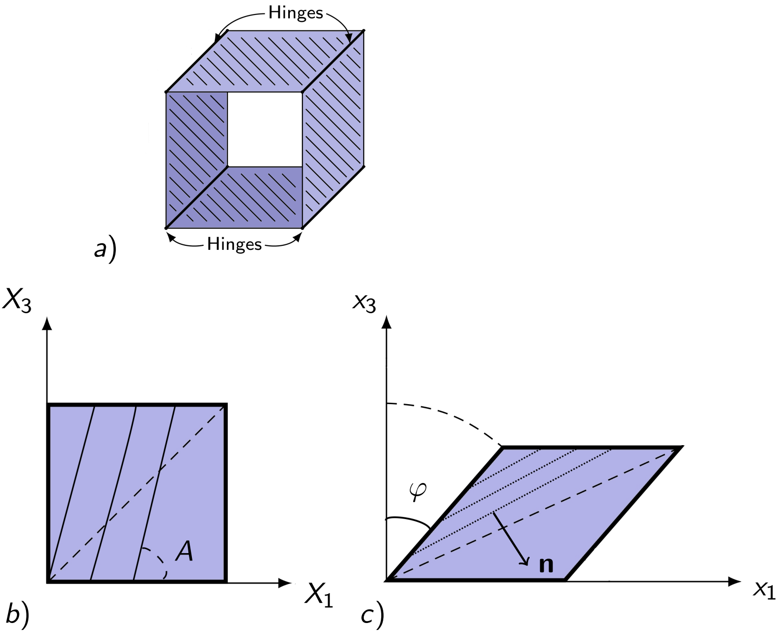


a)

Figure: A schematic of the shear-box deformation.


Hinges
Hinges
a)
X_3
X_1
b)
x_3
φ
x_1
c)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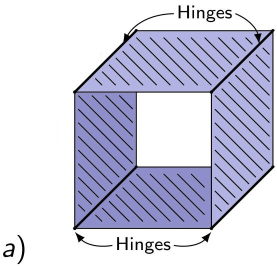
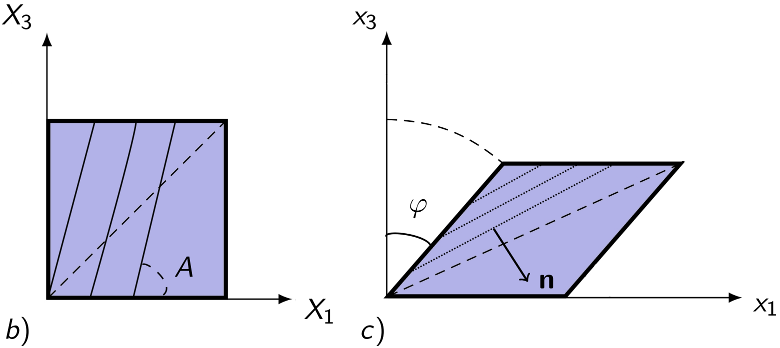


$$\mathbf{M} = (\cos A, 0, \sin A)_X$$

$$\mathbf{n} = (\cos \theta, 0, \sin \theta)_x$$



Surface Wrinkle Results

Incompressible $W = I_1 + \frac{A_S}{4}(I_4^S - 1)^2 + \frac{A_C}{4}(I_4^C - 1)^2$.

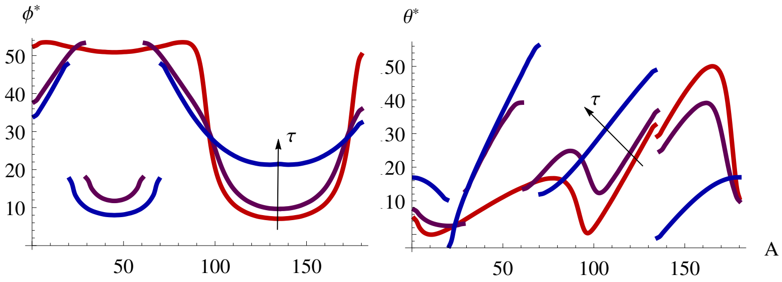


Figure: Critical deformation ϕ^* and wrinkle-front angle θ^* with $(A_S, A_C) = 16(\cos\tau, \sin\tau)$ and $\tau = 0°$, $45°$ and $90°$.

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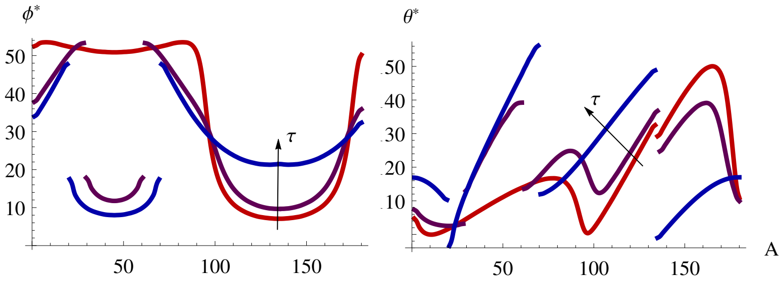


Figure: Critical deformation ϕ^* and wrinkle-front angle θ^* with $(A_S, A_C) = 16(\cos\tau, \sin\tau)$ and $\tau = 0^\circ$, 45° and 90°.

Might be simpler to investigate with either $A_S = 0$ or $A_C = 0$.

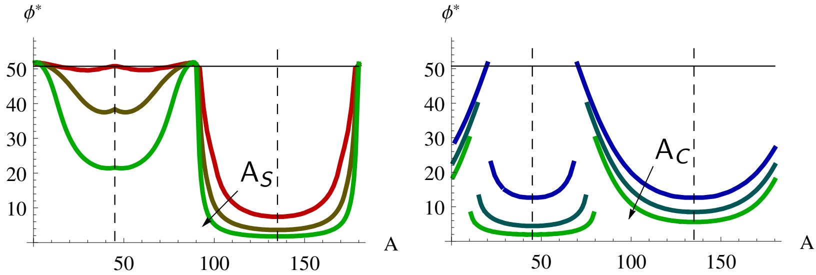
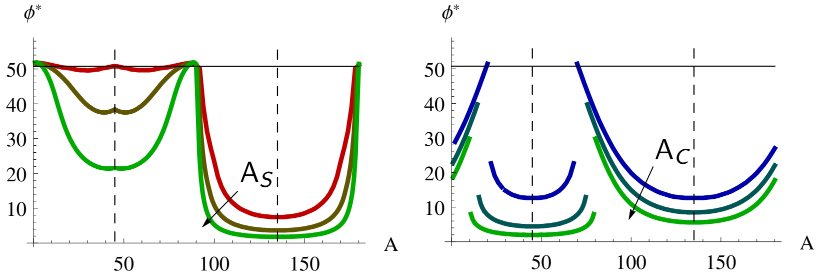


Figure: Both A_S and A_C take the values 16, 32 and 64. The solid black line $\phi^* = 50.75^\circ$ for $A_S = A_C = 0$.

To understand the wrinkle wavefront angle θ we need the angles α_S and α_C that

$$\mathbf{m}^S = \mathbf{F}\mathbf{M} \quad \text{and} \quad \mathbf{m}^C = \mathbf{F}^{-T}\mathbf{M}$$

make with the x_1−axis.



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make with the x_1-axis. ($I_4^S = \|\mathbf{m}^S\|^2$ and $I_4^C = \|\mathbf{m}^C\|^2$)

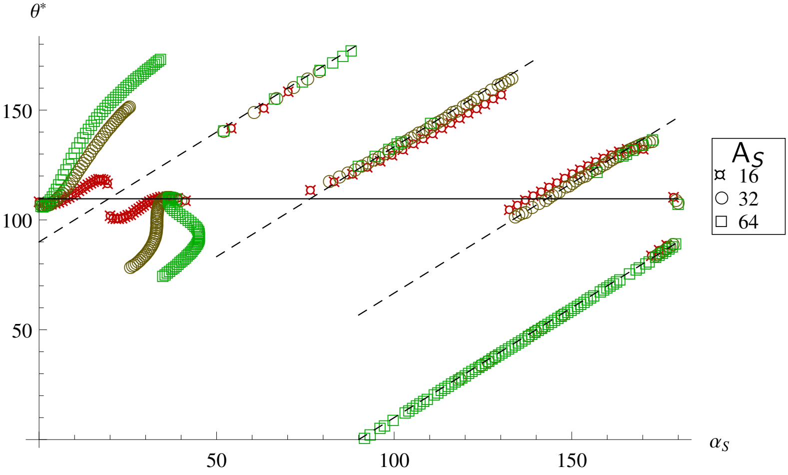
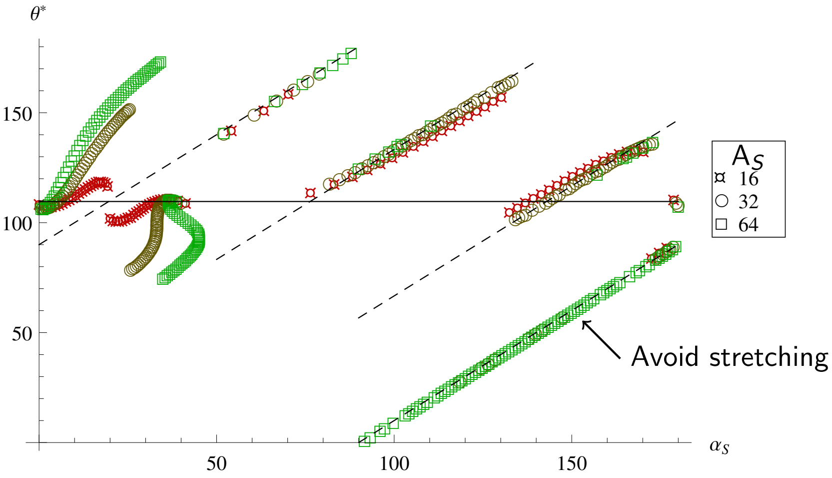


Figure: The dashed lines are either $\theta^* - \alpha_S = 90°$, $33.3°$ or $-33.3°$. The solid black line is given by $\theta^* = 109.6°$ and is the wrinkle-front angle if there were no fibres.



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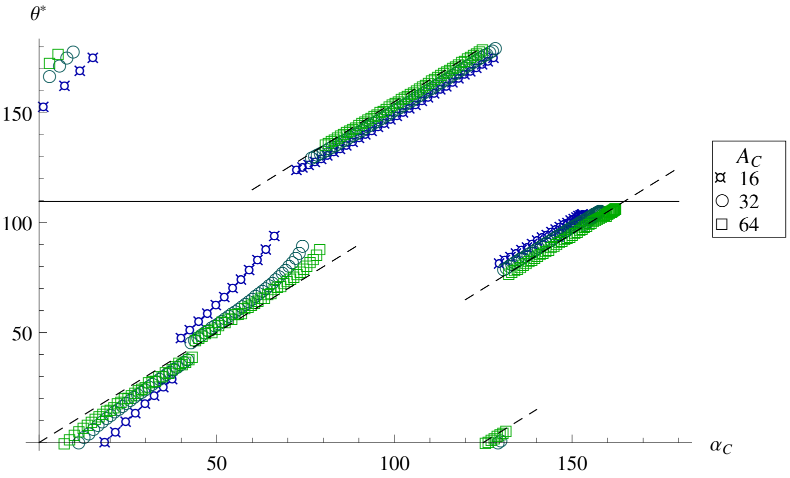
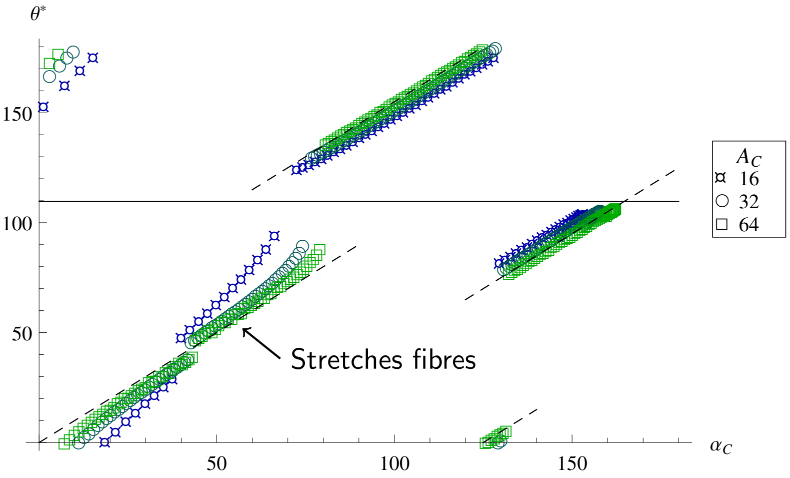


Figure: The dashed lines are either $\theta^* - \alpha_S = 0°$, $55°$ or $-55°$. The solid black line is given by $\theta^* = 109.6°$ and is the wrinkle-front angle if there were no fibres.



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Adding nonlinearity (Mooney-Rivlin) to the soft matrix does alter the phenomena. How about $A_S = A_C = 32$,

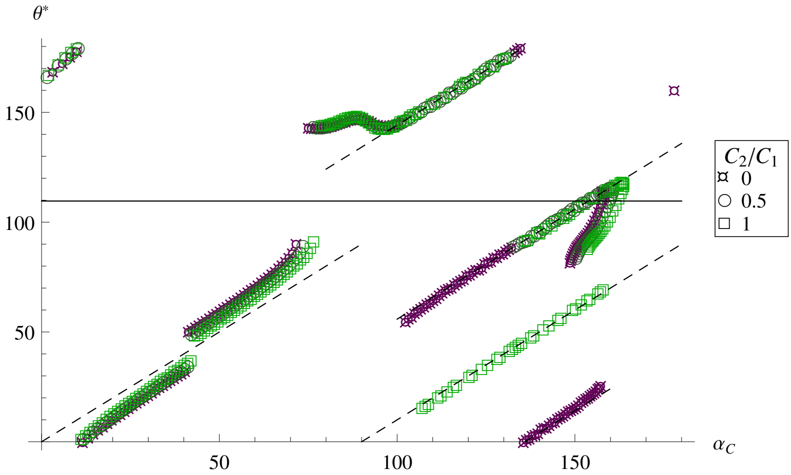

Figure: The dashed lines are either $\theta^* - \alpha_C = 0°$, $90°$, $33.3°/2 + 55°/2$ or $-33.3°/2 - 55°/2$.

To what extent does this *quanta* phenomena occur? Let's run some experiments for

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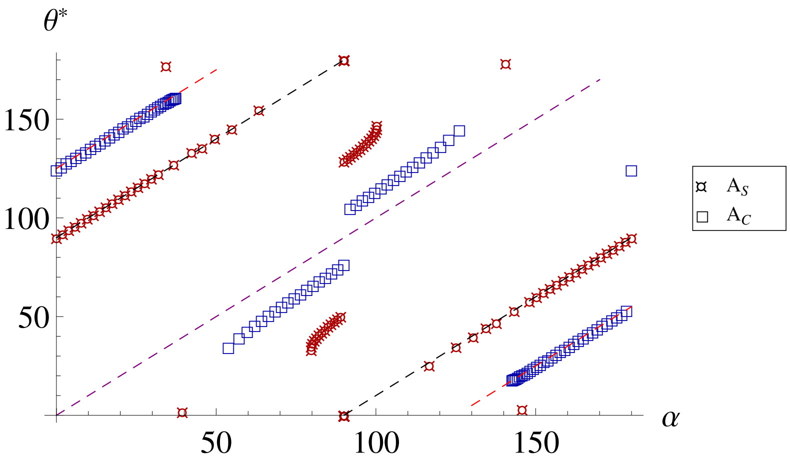
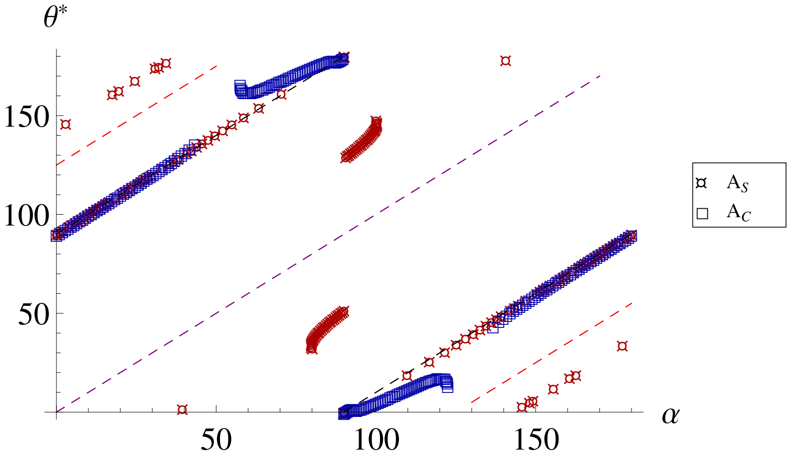


Red: $(A_S, A_C) = (32, 0)$, $\alpha = \alpha_S$,
Blue: $(A_S, A_C) = (0, 32)$, $\alpha = \alpha_C$ with surface stress.

To what extent does this *quanta* phenomena occur? Let's run some experiments for
- non-zero surface stress
- using I_5 in place of I_4^C



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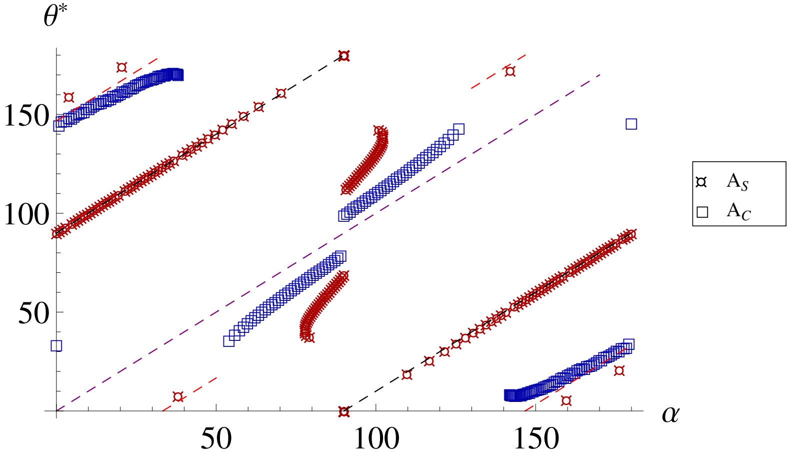
To what extent does this *quanta* phenomena occur? Let's run some experiments for ■ non-zero surface stress ■ using I_5 in place of I_4^C ■ remove incompressibility



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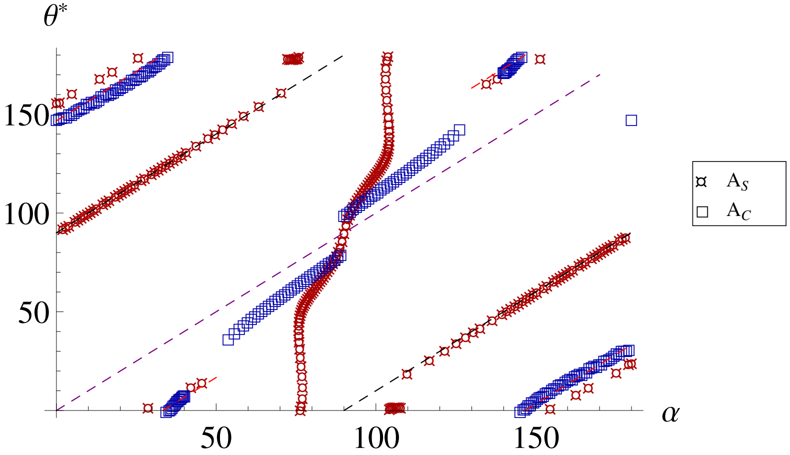
To what extent does this *quanta* phenomena occur? Let's run some experiments for

- non-zero surface stress
- using I_5 in place of I_4^C
- remove incompressibility
- only planar



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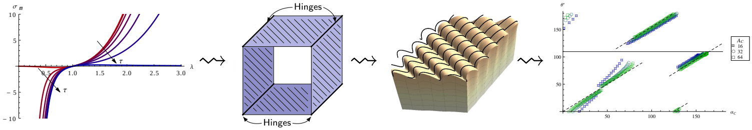

Any questions?

Thanks for listening and I hope you enjoyed the talk!

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