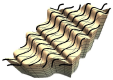
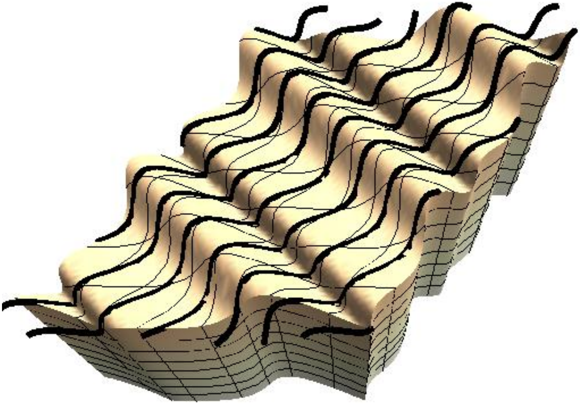


Wrinkling Anisotropy

Author:
Artur L. Gower

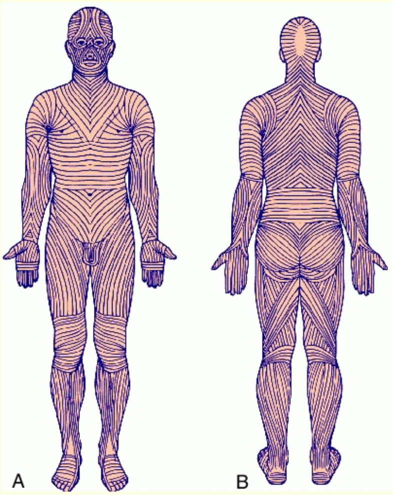
Co-Authors:
Prof. Michel Destrade
Dr. Pasquale Ciarletta

National University of Ireland Galway



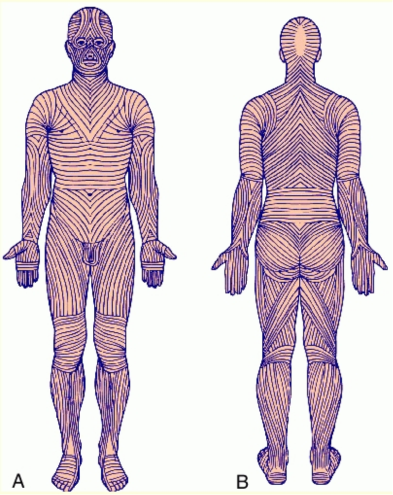
Gif showing a wrinkle appear and be sustained.

What do the wrinkles tell us?



⇝ The Langer-Lines are collagen fibers.

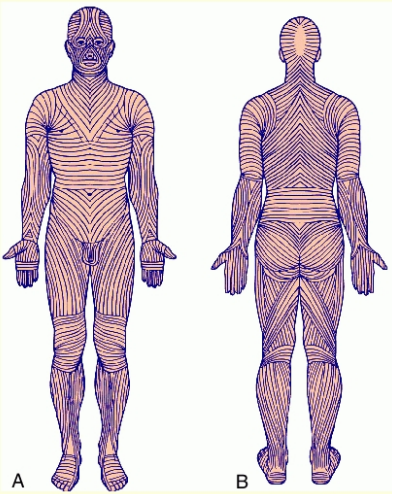
What do the wrinkles tell us?



⇝ The Langer-Lines are collagen fibers.

⇝ Incisions made parallel to Langer's lines produce less scarring.

What do the wrinkles tell us?



⇝ The Langer-Lines are collagen fibers.

⇝ Incisions made parallel to Langer's lines produce less scarring.

⇝ The exact direction of the collagen fibers are unknown.

What do the wrinkles tell us?

$\rightsquigarrow$ The pinch test.

What do the wrinkles tell us?

⇝ The pinch test.

⇝ Wrinkles identify fibre orientation.

What do the wrinkles tell us?

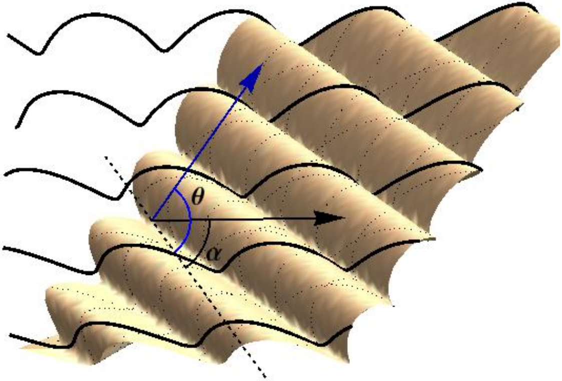
⇝ The pinch test.

⇝ Wrinkle prevention (temper rolling)

⇝ Wrinkles identify fibre orientation.

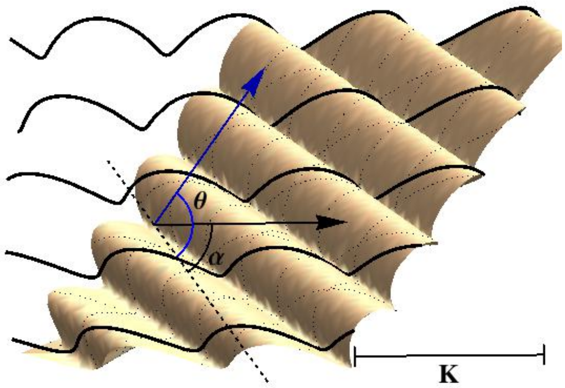
We look for solutions

$$\mathbf{u}(x, y, \theta) = \mathbf{U}(y)\mathrm{e}^{\mathrm{i}k(x\cos\theta + y\sin\theta)},$$

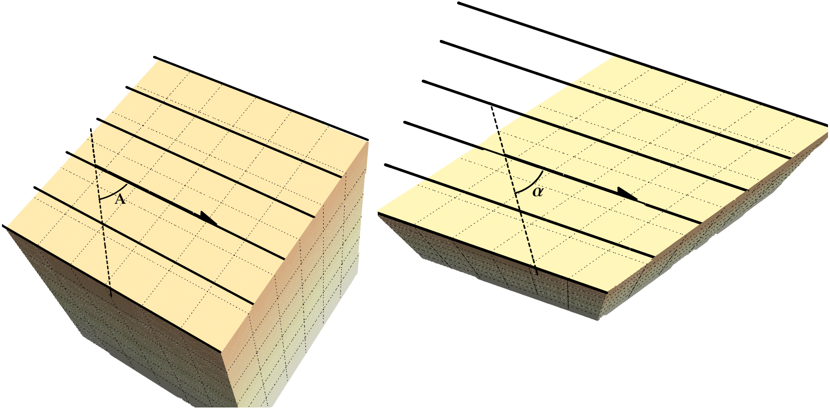


We look for solutions

$$\mathbf{u}(x, y, \theta) = \mathbf{U}(y)\mathrm{e}^{\mathrm{i}k(x\cos\theta + y\sin\theta)},$$



$$\mathbf{M} = (\cos A, \sin A, 0), \quad \mathbf{m} = \mathbf{F}\mathbf{M} = \|\mathbf{m}\|(\cos \alpha, \sin \alpha, 0),$$



The Matrix Impedance Method

$$\mathbf{u} = \underbrace{\mathrm{e}^{\mathrm{i}k\mathbf{E}z}\mathbf{U}_0\mathrm{e}^{\mathrm{i}k(x\cos\theta + y\sin\theta)}}_{\text{Displacement}}, \quad \boldsymbol{\sigma}\cdot\mathbf{e}_z = \underbrace{-k\mathbf{Z}\mathrm{e}^{\mathrm{i}k\mathbf{E}z}\mathbf{U}_0\mathrm{e}^{\mathrm{i}k(x\cos\theta + y\sin\theta)}}_{\text{Normal Traction}},$$

with $\mathbf{E} = \mathbf{T}^{-1}[\theta, \alpha, K](\mathrm{i}\mathbf{Z} - \mathbf{R}[\theta, \alpha, K])$.

The Matrix Impedance Method

$$\mathbf{u} = \underbrace{\mathrm{e}^{\mathrm{i}k\mathbf{E}z}\mathbf{U}_0\mathrm{e}^{\mathrm{i}k(x\cos\theta+y\sin\theta)}}_{\text{Displacement}}, \quad \boldsymbol{\sigma}\cdot\mathbf{e}_z = \underbrace{-k\mathbf{Z}\mathrm{e}^{\mathrm{i}k\mathbf{E}z}\mathbf{U}_0\mathrm{e}^{\mathrm{i}k(x\cos\theta+y\sin\theta)}}_{\text{Normal Traction}},$$

with $\mathbf{E} = \mathbf{T}^{-1}[\theta,\alpha,K](\mathrm{i}\mathbf{Z} - \mathbf{R}[\theta,\alpha,K])$.

$\rightsquigarrow$ Plug into div $\boldsymbol{\sigma} = 0$, resulting in

$$\mathbf{Q}[\theta,A,K] - \mathbf{H}^{\dagger}[\mathbf{Z}]\mathbf{H}[\mathbf{Z}] = 0,$$

with

$$\mathbf{H}[\mathbf{Z}] = \mathbf{T}[\theta,A,K]^{-1/2}(\mathbf{Z} + \mathrm{i}\mathbf{R}[\theta,A,K]),$$

The Matrix Impedance Method

$$\mathbf{u} = \underbrace{\mathrm{e}^{\mathrm{i}k\mathbf{E}z}\mathbf{U}_0\mathrm{e}^{\mathrm{i}k(x\cos\theta + y\sin\theta)}}_{\text{Displacement}}, \quad \boldsymbol{\sigma}\cdot\mathbf{e}_z = \underbrace{-k\mathbf{Z}\mathrm{e}^{\mathrm{i}k\mathbf{E}z}\mathbf{U}_0\mathrm{e}^{\mathrm{i}k(x\cos\theta + y\sin\theta)}}_{\text{Normal Traction}},$$

with $\mathbf{E} = \mathbf{T}^{-1}[\theta, \alpha, K](\mathrm{i}\mathbf{Z} - \mathbf{R}[\theta, \alpha, K])$.

$\rightsquigarrow$ Plug into div $\boldsymbol{\sigma} = 0$, resulting in

$$\mathbf{Q}[\theta, A, K] - \mathbf{H}^{\dagger}[\mathbf{Z}]\mathbf{H}[\mathbf{Z}] = 0,$$

with

$$\mathbf{H}[\mathbf{Z}] = \mathbf{T}[\theta, A, K]^{-1/2}(\mathbf{Z} + \mathrm{i}\mathbf{R}[\theta, A, K]),$$

and correct decay condition $\mathbf{Z} > 0$. The matrices $\mathbf{Q}, \mathbf{T}$ and $\mathbf{R}$ depend on K, θ and A .

The Matrix Impedance Method

$$\mathbf{u} = \underbrace{\mathrm{e}^{\mathrm{i}k\mathbf{E}z}\mathbf{U}_0\mathrm{e}^{\mathrm{i}k(x\cos\theta + y\sin\theta)}}_{\text{Displacement}}, \quad \boldsymbol{\sigma}\cdot\mathbf{e}_z = \underbrace{-k\mathbf{Z}\mathrm{e}^{\mathrm{i}k\mathbf{E}z}\mathbf{U}_0\mathrm{e}^{\mathrm{i}k(x\cos\theta + y\sin\theta)}}_{\text{Normal Traction}},$$

with $\mathbf{E} = \mathbf{T}^{-1}[\theta, \alpha, K](\mathrm{i}\mathbf{Z} - \mathbf{R}[\theta, \alpha, K])$.

$\rightsquigarrow$ Plug into div $\boldsymbol{\sigma} = 0$, resulting in

$$\mathbf{Q}[\theta, A, K] - \mathbf{H}^{\dagger}[\mathbf{Z}]\mathbf{H}[\mathbf{Z}] = 0,$$

with

$$\mathbf{H}[\mathbf{Z}] = \mathbf{T}[\theta, A, K]^{-1/2}(\mathbf{Z} + \mathrm{i}\mathbf{R}[\theta, A, K]),$$

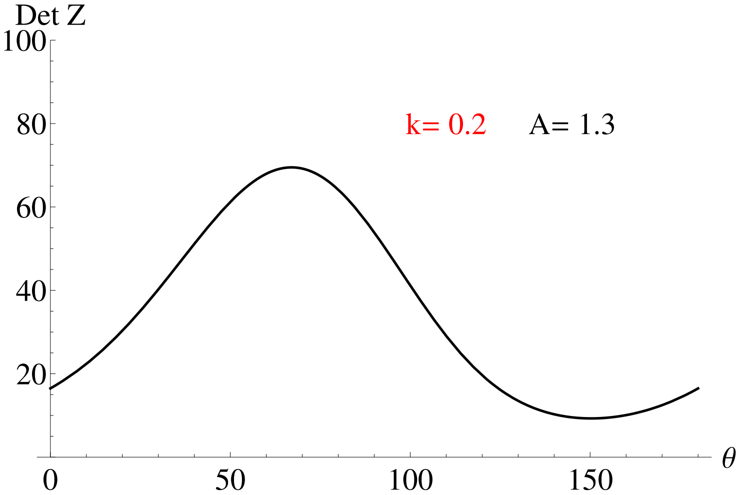
and correct decay condition $\mathbf{Z} > 0$. The matrices $\mathbf{Q}, \mathbf{T}$ and $\mathbf{R}$ depend on K, θ and A .

$\rightsquigarrow$ Zero surface-traction

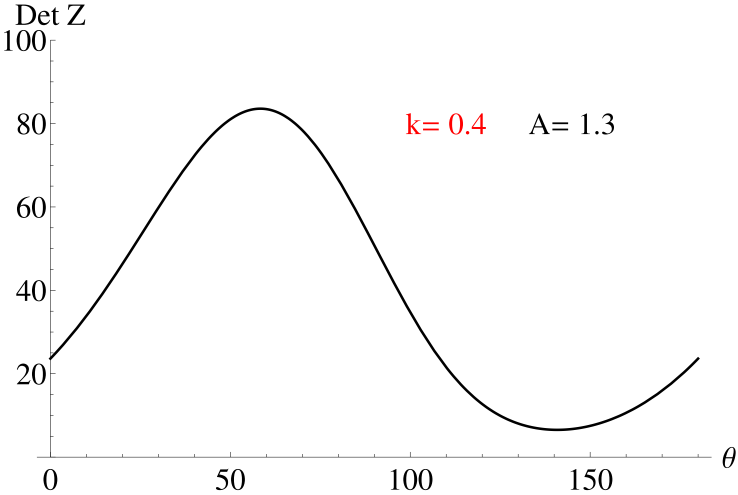
$$\boldsymbol{\sigma} \cdot \mathbf{e}_z\Big|_{z=0} = 0 \implies \mathbf{Z}\mathbf{U}_0 = 0 \implies \det \mathbf{Z} = 0.$$

The method is to fix A, and then for each K: *Open Gifs*

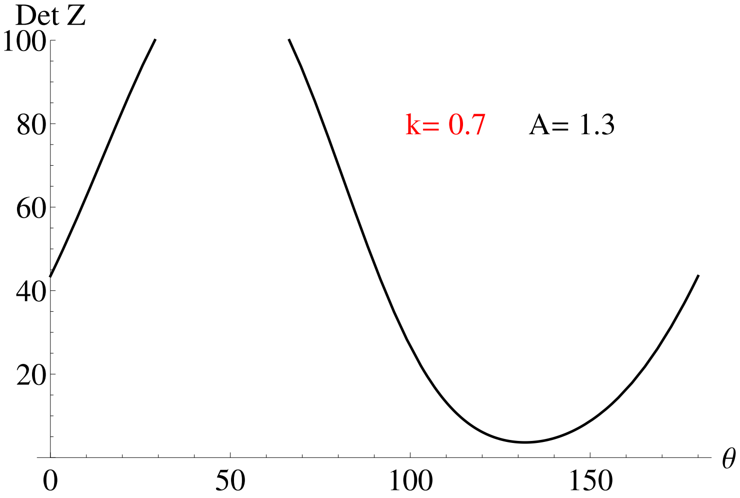
The method is to fix A, and then for each K:



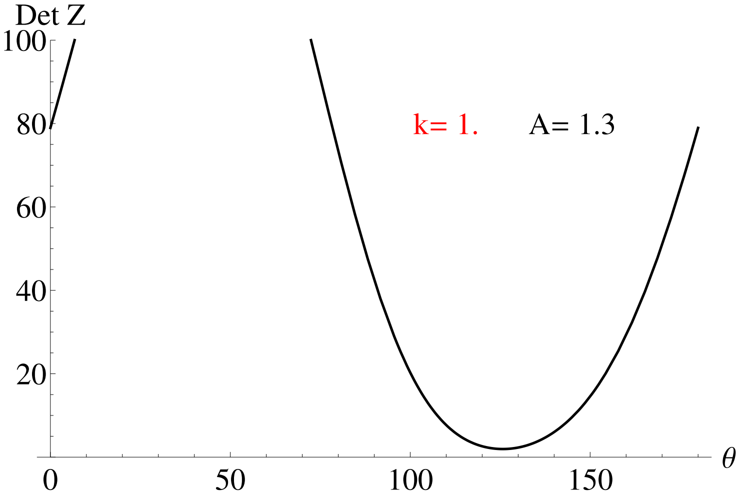
The method is to fix A, and then for each K:



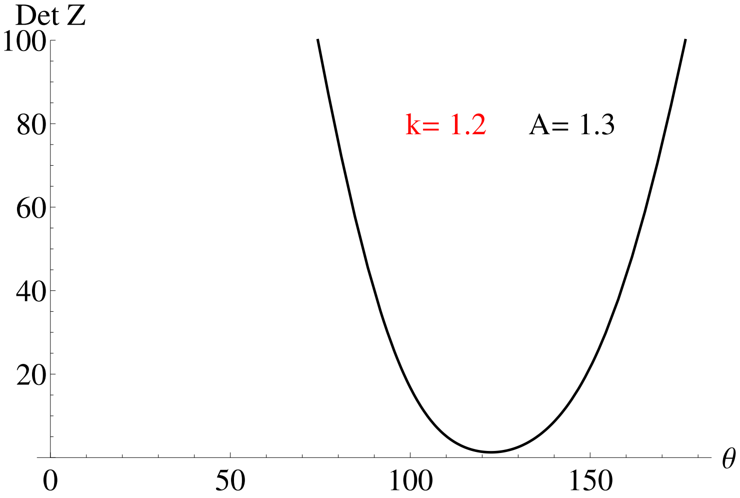
The method is to fix A, and then for each K:



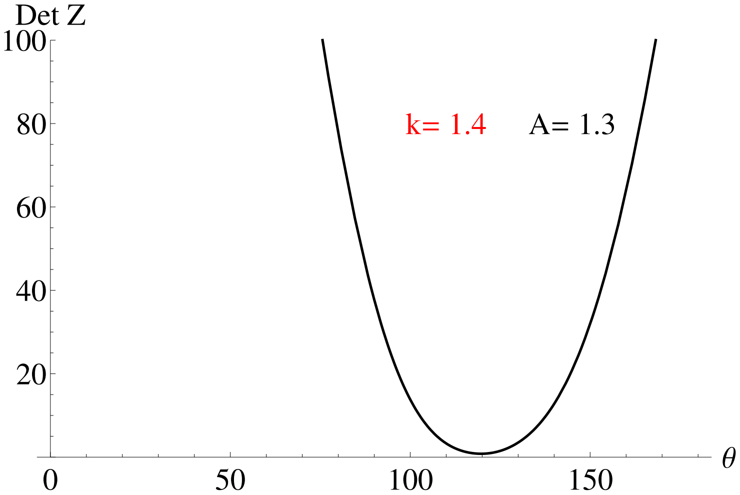
The method is to fix A, and then for each K:



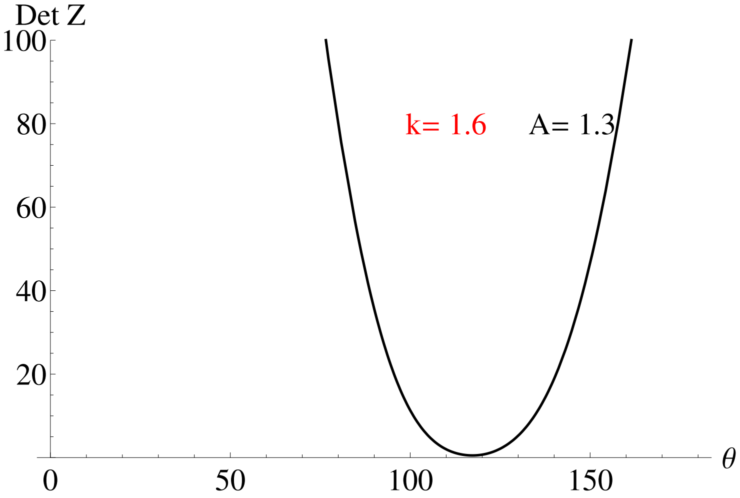
The method is to fix A, and then for each K:



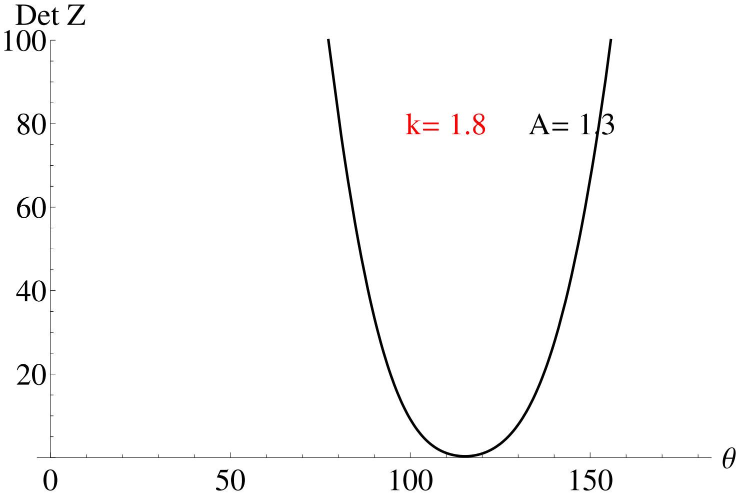
The method is to fix A, and then for each K:



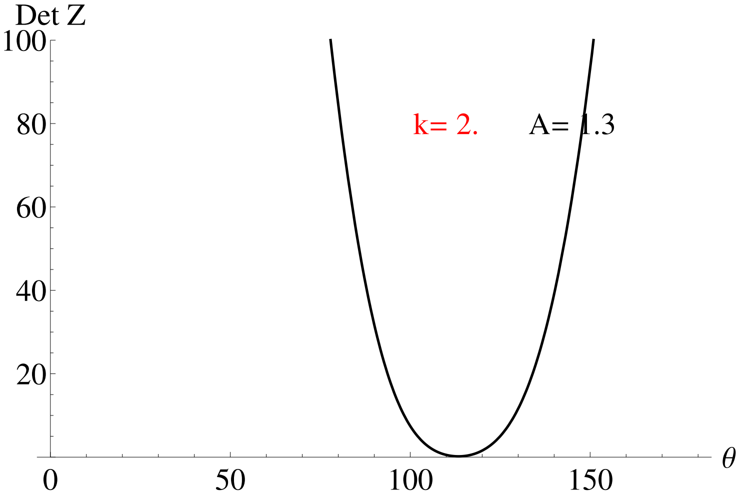
The method is to fix A, and then for each K:



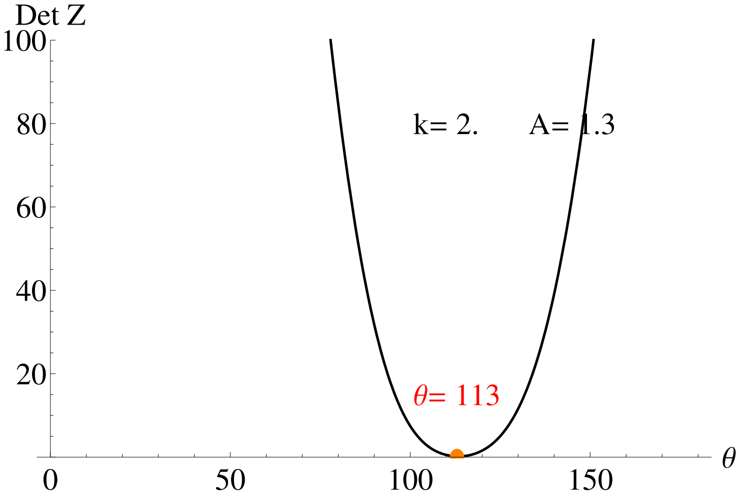
The method is to fix A, and then for each K:



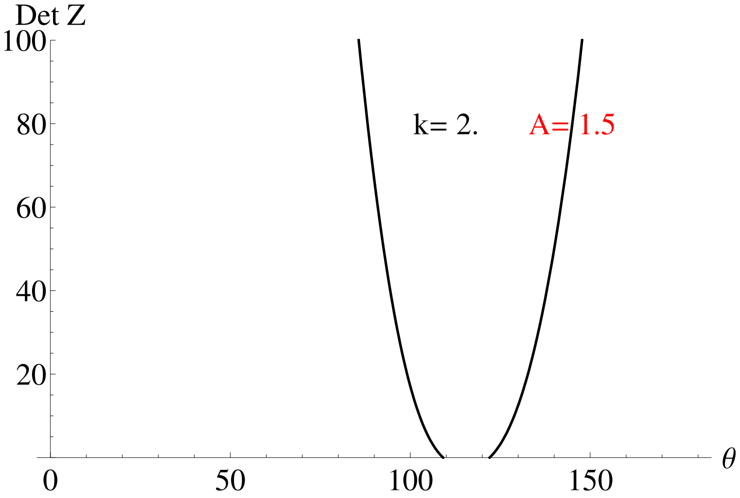
The method is to fix A, and then for each K:



The method is to fix A, and then for each K:



The method is to fix A, and then for each K:



Beware if you miss the critical K_{cr}

Beware if you miss the critical K_{cr}

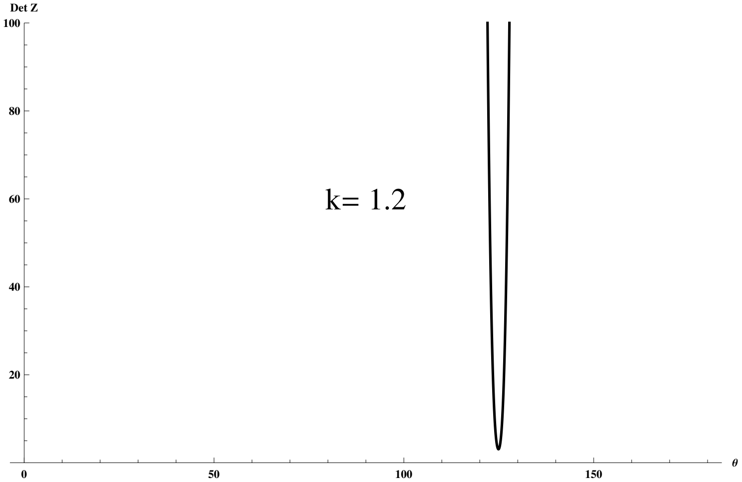


Figure: Fibres $\beta_+/\mu = 20$, resist only extension.

Beware if you miss the critical K_{cr}

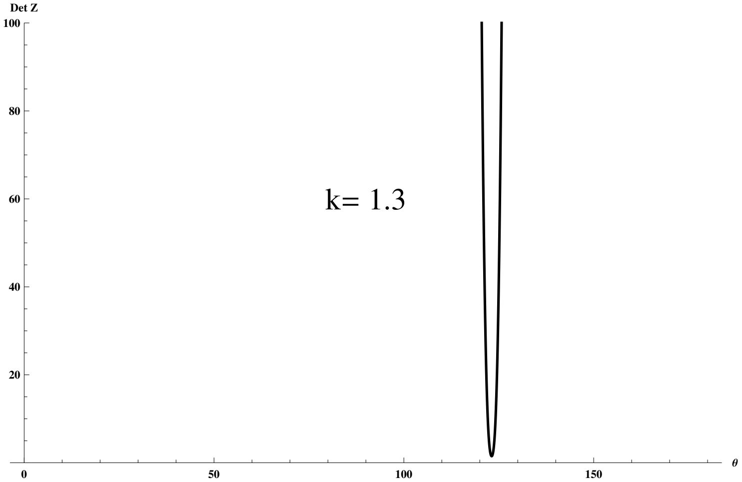


Figure: Fibres $\beta_+/\mu = 20$, resist only extension.

Beware if you miss the critical K_{cr}

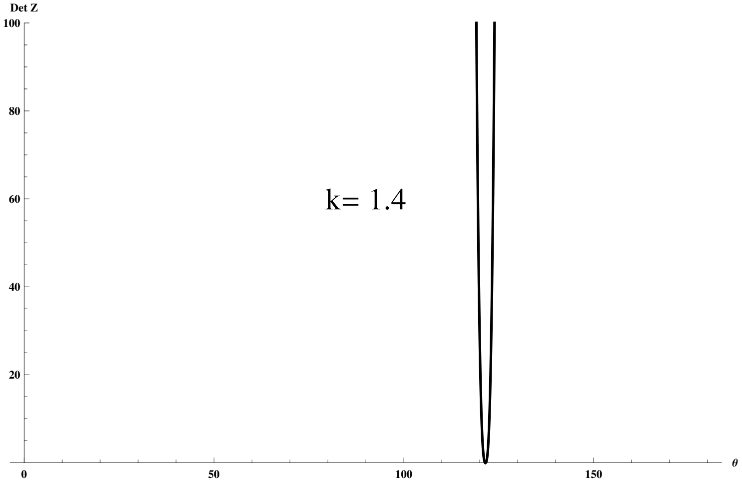


Figure: Fibres $\beta_+/\mu = 20$, resist only extension.

Beware if you miss the critical K_{cr}

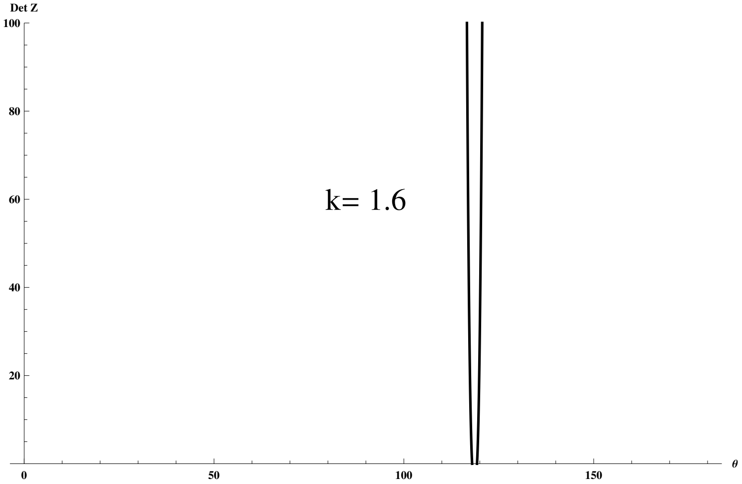


Figure: Fibres $\beta_+/\mu = 20$, resist only extension.

Beware if you miss the critical K_{cr}

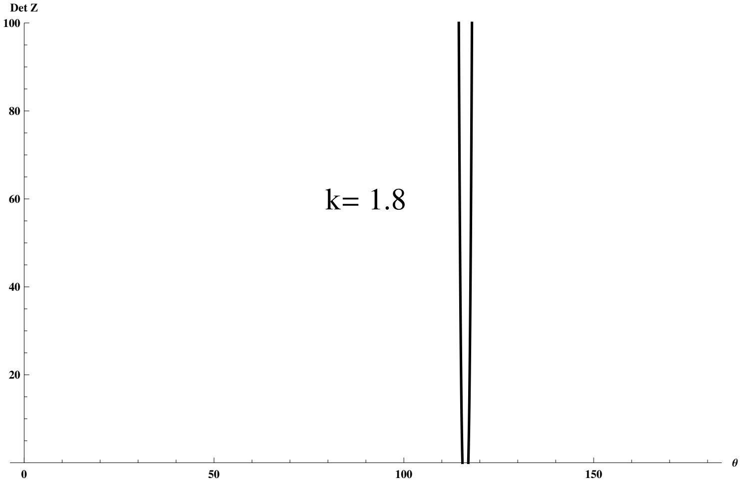


Figure: Fibres $\beta_+/\mu = 20$, resist only extension.

A simple choice with a range of anisotropy

$$W = \frac{\mu}{2}(\mathrm{tr}\ \mathbf{C} - 3) + f(\det\ \mathbf{C})$$
$$+ \frac{\beta_+}{4}(\mathbf{M}\cdot\mathbf{C}\mathbf{M} - 1)^2 + \frac{\beta_-}{4}(\mathbf{M}\cdot\mathbf{C}^{-1}\mathbf{M} - 1)^2$$

A simple choice with a range of anisotropy

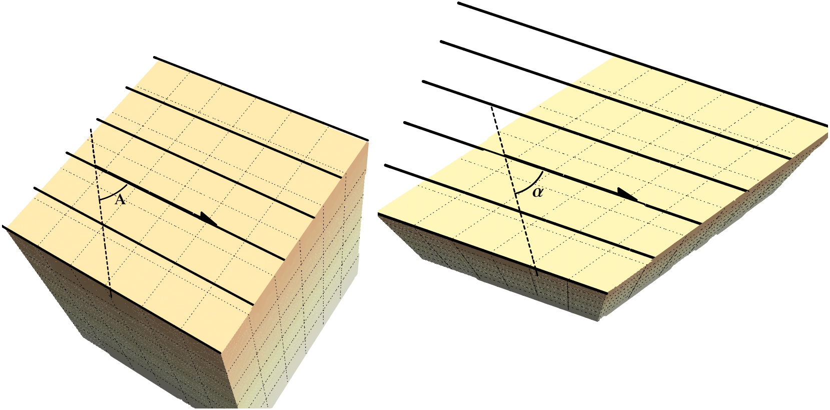
$$W = \frac{\mu}{2}(\mathrm{tr}\ \mathbf{C} - 3) + f(\det\ \mathbf{C}) + \frac{\beta_+}{4}\underbrace{(\mathbf{M}\cdot\mathbf{C}\mathbf{M} - 1)^2}_{\text{Extension}} + \frac{\beta_-}{4}(\mathbf{M}\cdot\mathbf{C}^{-1}\mathbf{M} - 1)^2$$

A simple choice with a range of anisotropy

$$W = \frac{\mu}{2}(\mathrm{tr}\ \mathbf{C} - 3) + f(\det\ \mathbf{C}) + \frac{\beta_+}{4} \underbrace{(\mathbf{M} \cdot \mathbf{C}\mathbf{M} - 1)^2}_{\text{Extension}} + \frac{\beta_-}{4} \underbrace{(\mathbf{M} \cdot \mathbf{C}^{-1}\mathbf{M} - 1)^2}_{\text{Compression}}$$

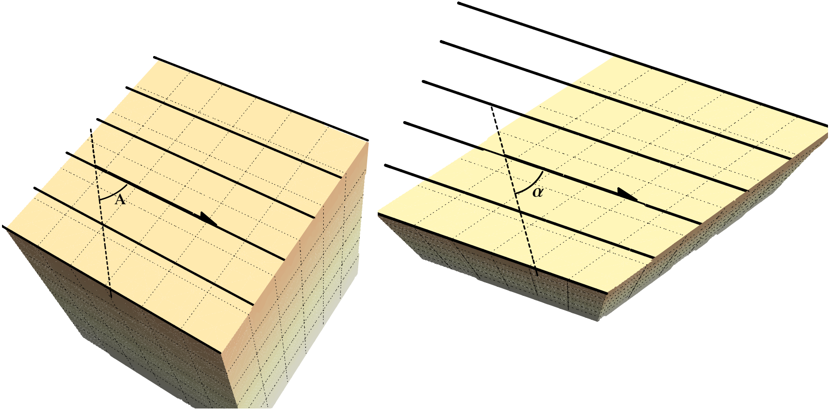
A simple choice with a range of anisotropy

$$W = \frac{\mu}{2}(\mathrm{tr}\ \mathbf{C} - 3) + f(\det\ \mathbf{C}) + \frac{\beta_+}{4}\underbrace{(\mathbf{M}\cdot\mathbf{C}\mathbf{M} - 1)^2}_{\text{Extension}} + \frac{\beta_-}{4}\underbrace{(\mathbf{M}\cdot\mathbf{C}^{-1}\mathbf{M} - 1)^2}_{\text{Compression}}$$

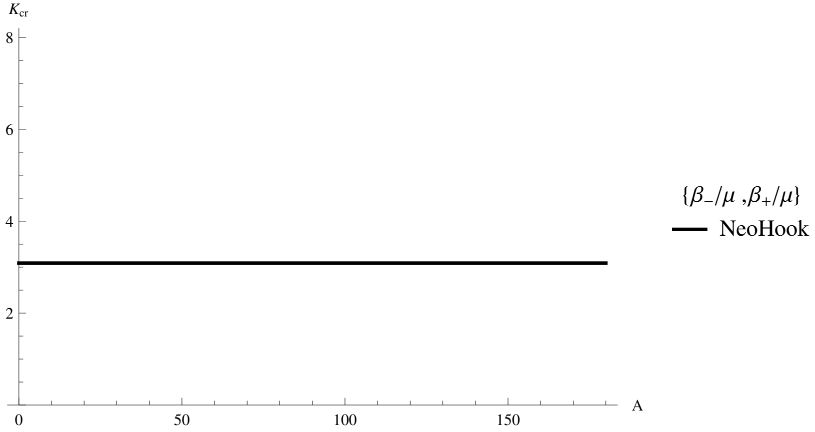


A simple choice with a range of anisotropy

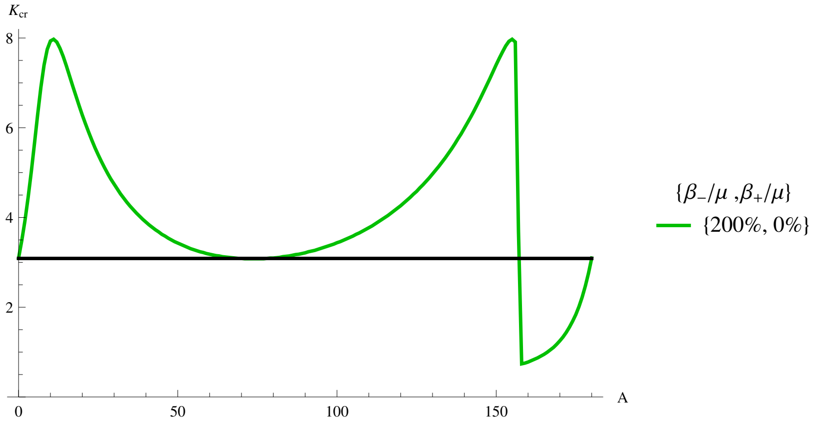
$$W = \frac{\mu}{2}(\mathrm{tr}\ \mathbf{C} - 3) + f(\det\ \mathbf{C}) + \frac{\beta_+}{4}\underbrace{(\mathbf{M}\cdot\mathbf{CM}-1)^2}_{\text{Extension}} + \frac{\beta_-}{4}\underbrace{(\mathbf{M}\cdot\mathbf{C}^{-1}\mathbf{M}-1)^2}_{\text{Compression}}$$



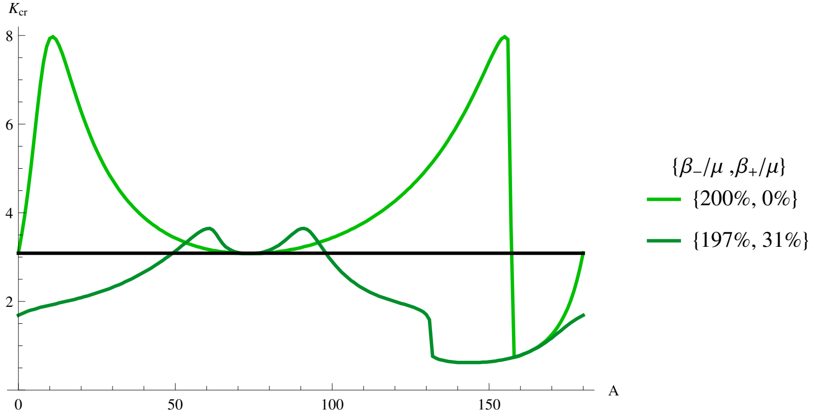
$$\mathbf{M} = (\cos A, \sin A, 0)$$



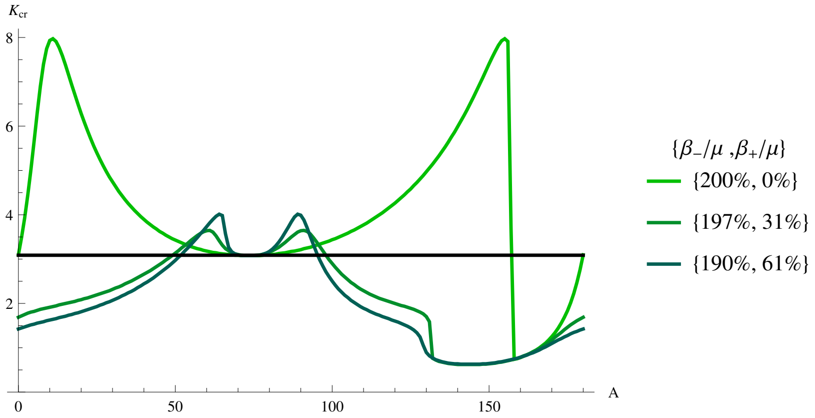
$$W_\beta = \frac{\beta_+}{4}(\mathbf{M}\cdot\mathbf{CM}-1)^2 + \frac{\beta_-}{4}(\mathbf{M}\cdot\mathbf{C}^{-1}\mathbf{M}-1)^2$$



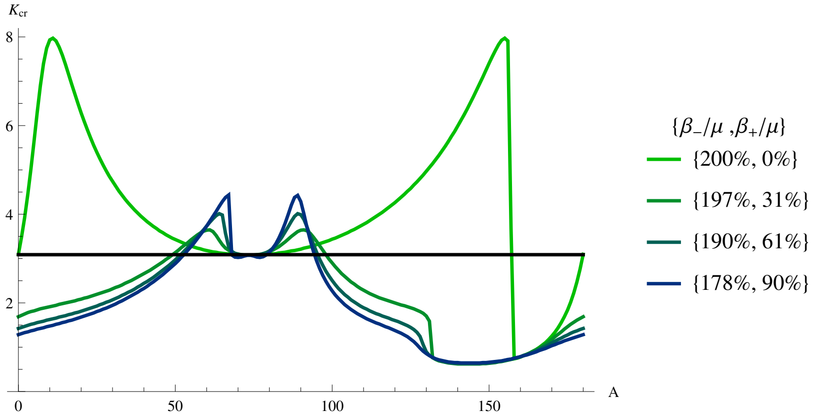
$$W_\beta = \frac{\beta_+}{4}(\mathbf{M} \cdot \mathbf{C}\mathbf{M} - 1)^2 + \frac{\beta_-}{4}(\mathbf{M} \cdot \mathbf{C}^{-1}\mathbf{M} - 1)^2$$



$$W_\beta = \frac{\beta_+}{4}(\mathbf{M}\cdot\mathbf{C}\mathbf{M}-1)^2 + \frac{\beta_-}{4}(\mathbf{M}\cdot\mathbf{C}^{-1}\mathbf{M}-1)^2$$



$$W_\beta = \frac{\beta_+}{4}(\mathbf{M}\cdot\mathbf{CM}-1)^2 + \frac{\beta_-}{4}(\mathbf{M}\cdot\mathbf{C}^{-1}\mathbf{M}-1)^2$$



$$W_\beta = \frac{\beta_+}{4}(\mathbf{M}\cdot\mathbf{C}\mathbf{M}-1)^2 + \frac{\beta_-}{4}(\mathbf{M}\cdot\mathbf{C}^{-1}\mathbf{M}-1)^2$$

Figure:

$- - - \ \alpha = \arctan \lambda_2(K)$, most stretched direction.

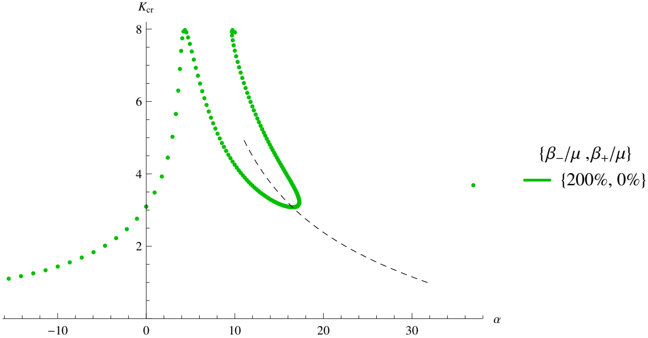


Figure:

$- - - \ \alpha = \theta_{cr} - 90^\circ$, Orthogonal to wrinkle.

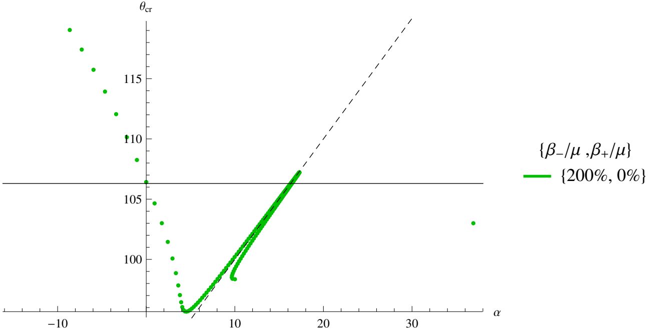


Figure:

- - - $\alpha = \arctan \lambda_2(K)$, most stretched direction.

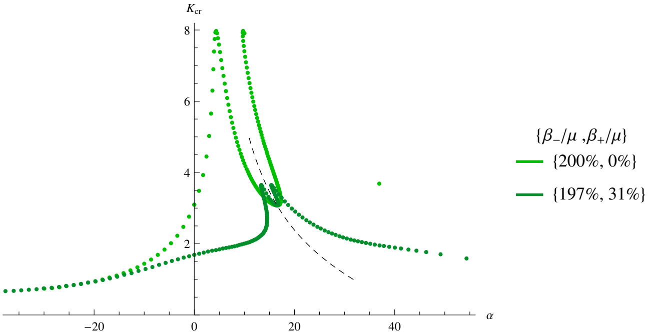


Figure:

- - - $\alpha = \theta_{cr} - 90^\circ$, Orthogonal to wrinkle.

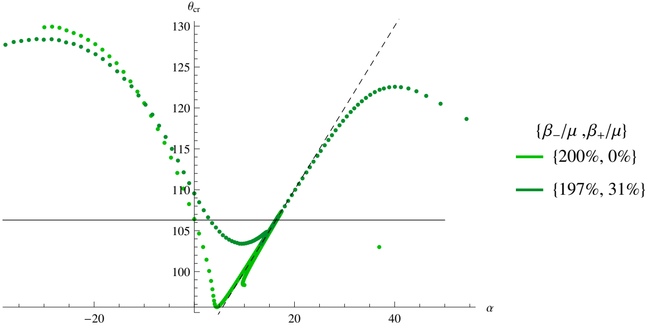


Figure:

- - - $\alpha = \arctan \lambda_2(K)$,

most stretched direction.

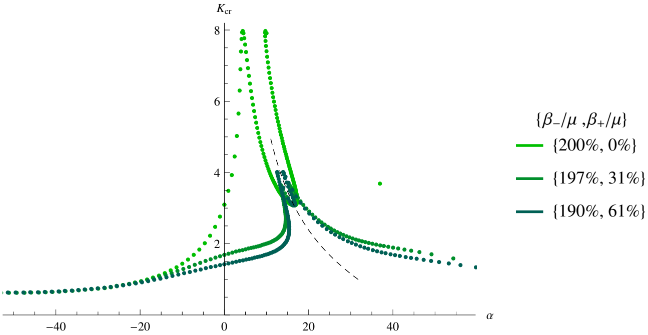


Figure:

- - - $\alpha = \theta_{cr} - 90^\circ$,

Orthogonal to wrinkle.

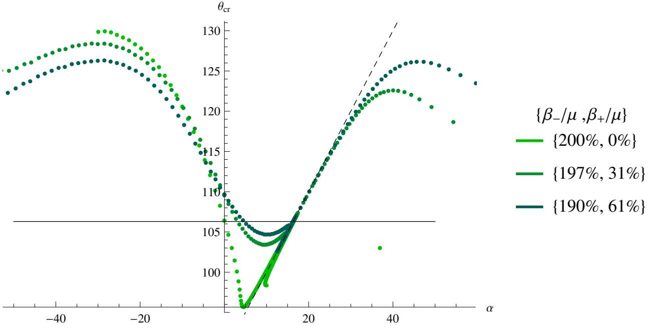


Figure: - - - $\alpha = \arctan \lambda_2(K)$, most stretched direction.

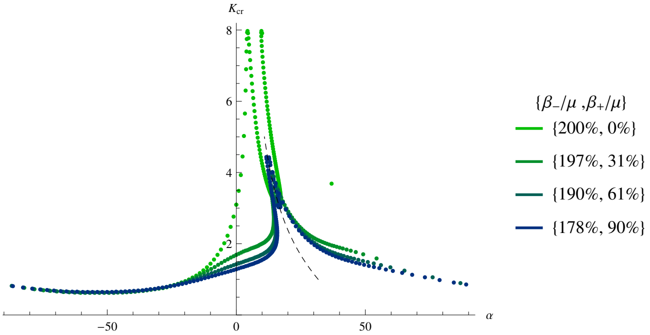


Figure: - - - $\alpha = \theta_{cr} - 90^\circ$, Orthogonal to wrinkle.

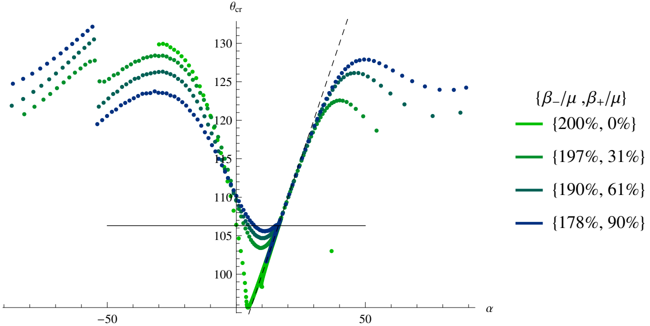


Figure:

- - - $\alpha = \arctan \lambda_2(K)$,

most stretched direction.

The stiff the fibres, the closer they hug these curves (experimental).

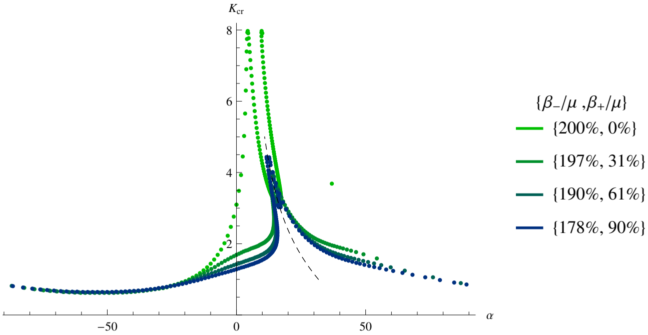
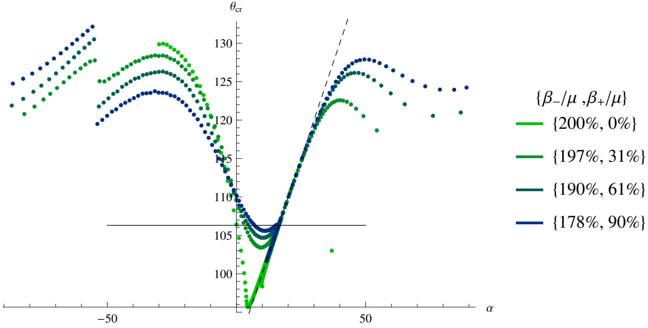


Figure:

- - - $\alpha = \theta_{cr} - 90^\circ$,

Orthogonal to wrinkle.



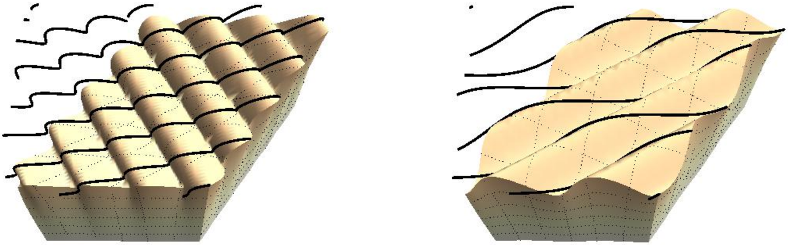
Given the wave vector $\mathbf{U}_0$ such that $\mathbf{Z}\mathbf{U}_0 = 0$, then

$$\mathbf{U}_0^\dagger(\mathbf{Q}-\mathbf{H}^\dagger\mathbf{H})\mathbf{U}_0 = 0 \implies \delta W(\mathbf{u})k^{-2} = \mathbf{U}_0^\dagger\mathbf{Q}\mathbf{U}_0 - \mathbf{U}_0^\dagger\mathbf{R}^T\mathbf{T}^{-1}\mathbf{R}\mathbf{U}_0 = 0,$$

Given the wave vector $\mathbf{U}_0$ such that $\mathbf{Z}\mathbf{U}_0 = 0$, then

$$\mathbf{U}_0^\dagger(\mathbf{Q} - \mathbf{H}^\dagger\mathbf{H})\mathbf{U}_0 = 0 \implies \delta W(\mathbf{u})k^{-2} = \mathbf{U}_0^\dagger\mathbf{Q}\mathbf{U}_0 - \mathbf{U}_0^\dagger\mathbf{R}^T\mathbf{T}^{-1}\mathbf{R}\mathbf{U}_0 = 0,$$

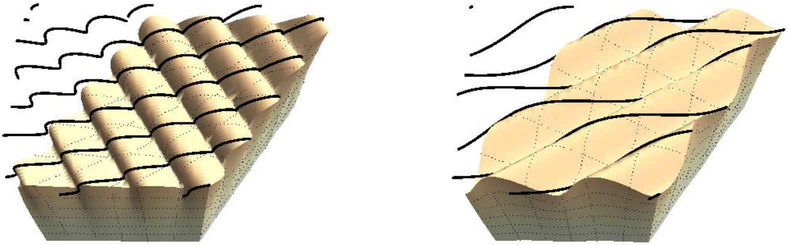
that is, zero-traction *implies* no (average density) potential energy increment.



Given the wave vector $\mathbf{U}_0$ such that $\mathbf{Z}\mathbf{U}_0 = 0$, then

$$\mathbf{U}_0^\dagger(\mathbf{Q}-\mathbf{H}^\dagger\mathbf{H})\mathbf{U}_0 = 0 \implies \delta W(\mathbf{u})k^{-2} = \mathbf{U}_0^\dagger\mathbf{Q}\mathbf{U}_0 - \mathbf{U}_0^\dagger\mathbf{R}^T\mathbf{T}^{-1}\mathbf{R}\mathbf{U}_0 = 0,$$

that is, zero-traction *implies* no (average density) potential energy increment.



$\rightsquigarrow$ More generally; the wrinkle will minimize δW.

Figure:

- - - $\alpha = \arctan \lambda_2(K)$,

most stretched direction.

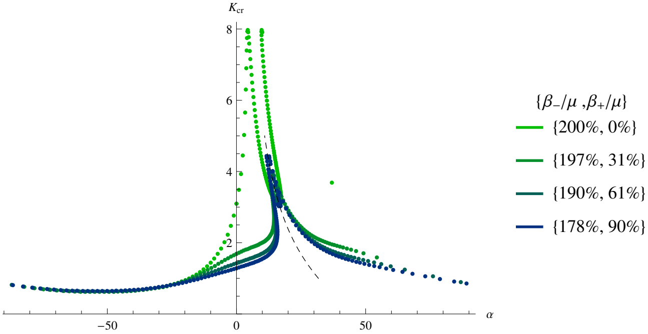


Figure:

- - - $\alpha = \theta_{cr} - 90°$,

Orthogonal to wrinkle.

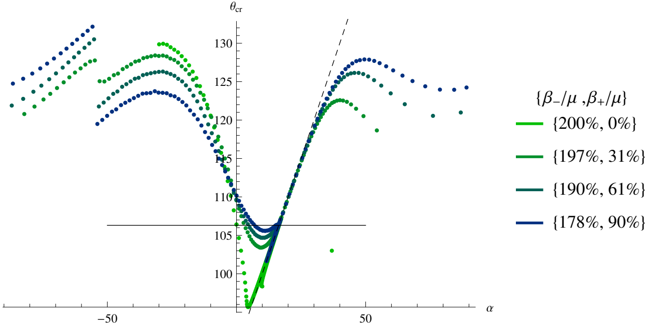
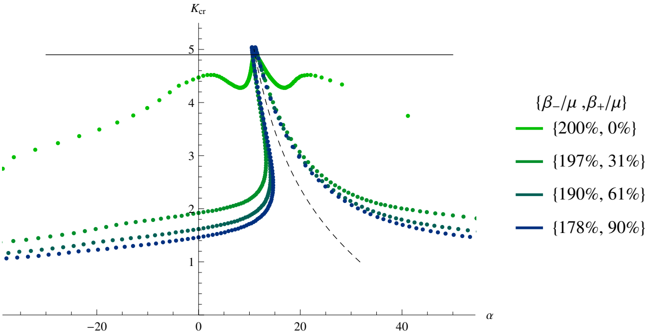


Figure:

$- - - \ \alpha = \arctan \lambda_2(K)$, most stretched direction.



$$W = W + \frac{\mu}{2 \det \mathbf{C}^{2/3}} \left(\mathrm{tr}\ \mathbf{C}^2 - (\mathrm{tr}\ \mathbf{C})^2 \right)$$

Figure:

$- - - \ \alpha = \theta_{cr} - 90^\circ$, Orthogonal to wrinkle.

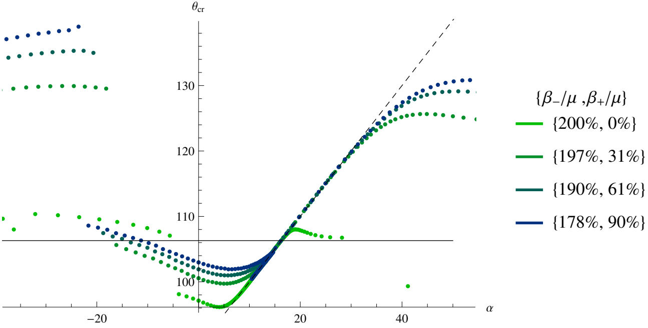
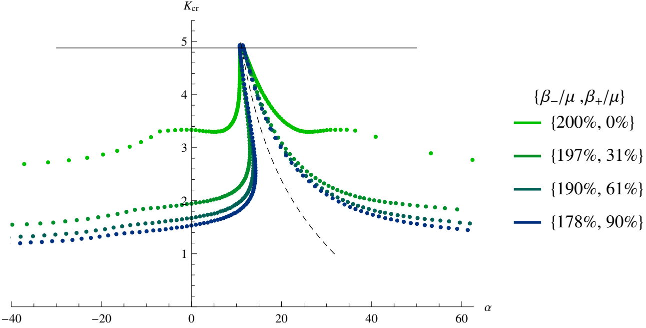


Figure:

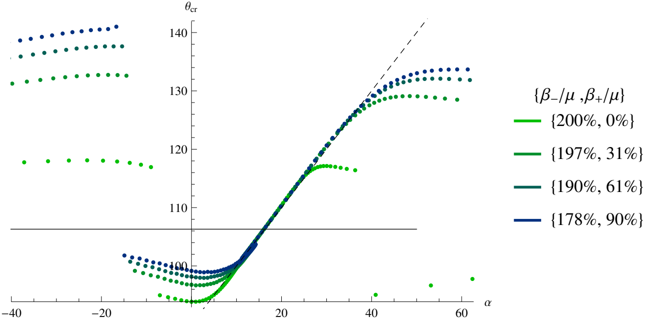
- - - $\alpha = \arctan \lambda_2(K)$, most stretched direction.



$$W = W + \frac{2\mu}{2 \det \mathbf{C}^{2/3}} (\mathrm{tr}\ \mathbf{C}^2 - (\mathrm{tr}\ \mathbf{C})^2)$$

Figure:

- - - $\alpha = \theta_{cr} - 90^\circ$, Orthogonal to wrinkle.



Lessons Learned

- Wrinkles can tend to align with Fibres, but ultimately leed to diverse behaviour.

Lessons Learned

- Wrinkles can tend to align with Fibres, but ultimately leed to diverse behaviour.
- Can the shared wrinkle point be related to how the underlying soft matrix wrinkles?

Lessons Learned

- Wrinkles can tend to align with Fibres, but ultimately leed to diverse behaviour.
- Can the shared wrinkle point be related to how the underlying soft matrix wrinkles?
- Do not use methods focused on solving $\det \mathbf{Z}(K) = 0$.

Lessons Learned

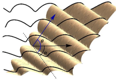
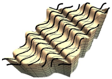
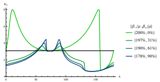
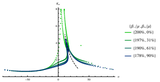
- Wrinkles can tend to align with Fibres, but ultimately leed to diverse behaviour.
- Can the shared wrinkle point be related to how the underlying soft matrix wrinkles?
- Do not use methods focused on solving $\det \mathbf{Z}(K) = 0$.
- The Riccati equation is numerically robust.

Lessons Learned

- Wrinkles can tend to align with Fibres, but ultimately leed to diverse behaviour.
- Can the shared wrinkle point be related to how the underlying soft matrix wrinkles?
- Do not use methods focused on solving $\det \mathbf{Z}(K) = 0$.
- The Riccati equation is numerically robust.
- Still much to explore.

Lessons Learned

- Wrinkles can tend to align with Fibres, but ultimately leed to diverse behaviour.
- Can the shared wrinkle point be related to how the underlying soft matrix wrinkles?
- Do not use methods focused on solving $\det \mathbf{Z}(K) = 0$.
- The Riccati equation is numerically robust.
- Still much to explore.

 $\rightsquigarrow$  $\rightsquigarrow$  $\rightsquigarrow$  $\rightsquigarrow$ 

Any questions?

Thanks for listening and hope you enjoyed the talk!

P. Ciarletta, M. Destrade, A.L. Gower., Shear instability in skin tissue, Quarterly Journal of Mechanics and Applied Mathematics, 66 (2013) 273-288.

A. Mielke, Y.B. Fu. A proof of uniqueness of surface waves that is independent of the Stroh Formalism, Math. Mech. Solids **9** (2003), 5–15.