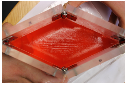


Oblique-Wrinkling

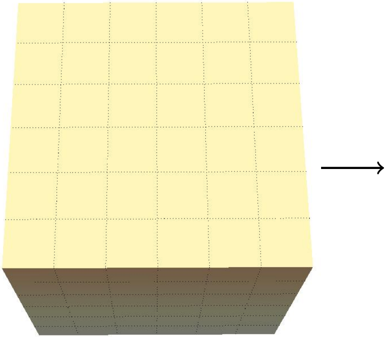
Author:
Artur L. Gower

Co-Author:
Prof. Michel Destrade

National University of Ireland Galway
NUI Galway
OÉ Gaillimh

A long long time ago...

Biot in 1963 predicts,



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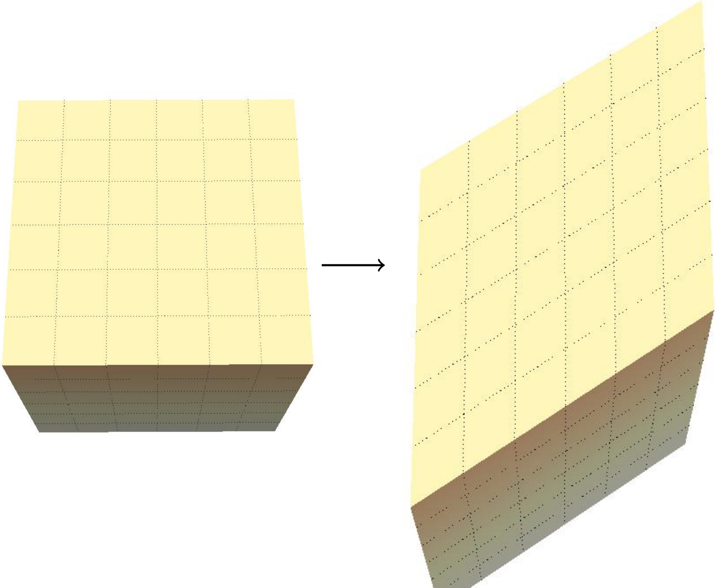

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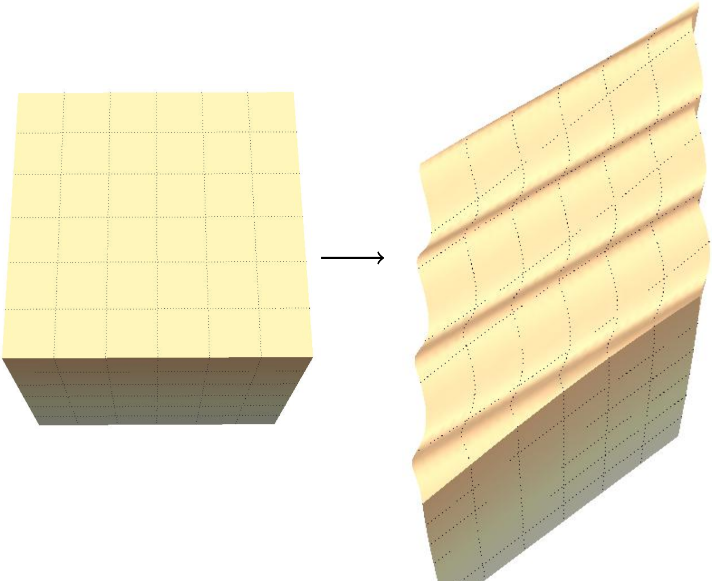

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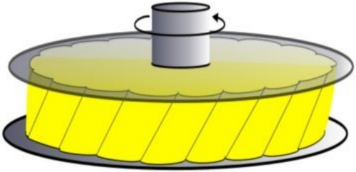

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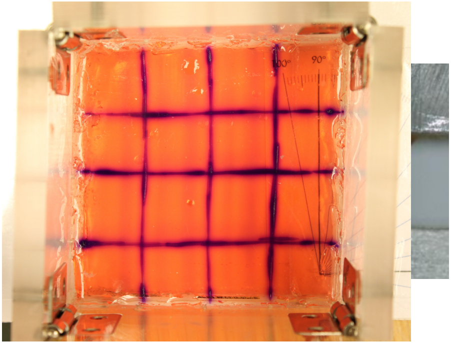

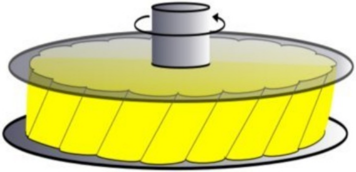

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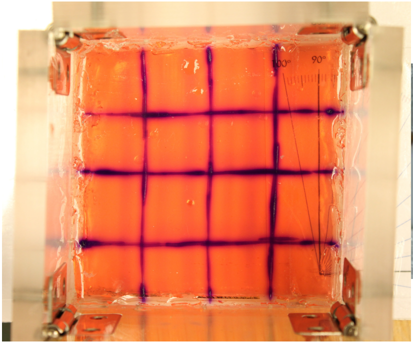

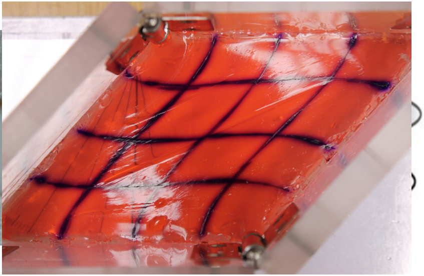

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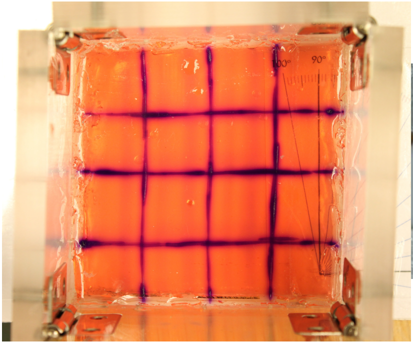
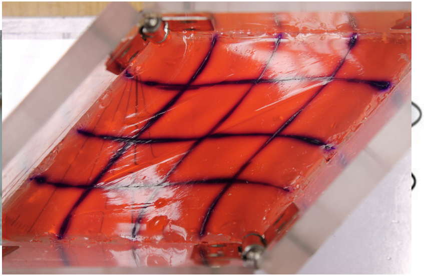
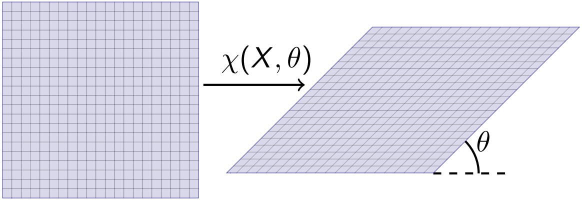




$\longrightarrow$ Maintains a more homogeneous deformation $\longrightarrow$

Theoretical Model

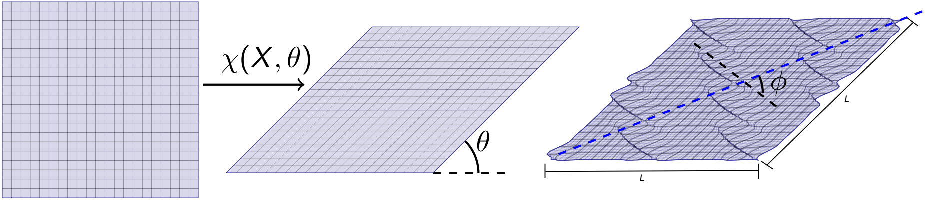
Shear-box $x = \chi(X, \theta)$



Theoretical Model

Shear-box $x = \chi(X, \theta)$ plus small $\tilde{x}_j = x_j + u_j$ with

$$u_j(x, y, z) = U_j(y) e^{ik(x \cos\phi + z \sin\phi)}.$$



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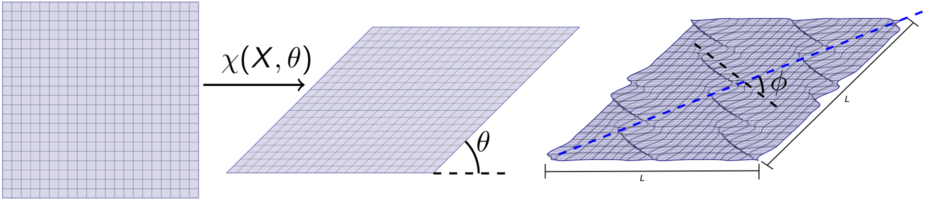

$$\underbrace{\text{div}\, \boldsymbol{\sigma} = 0 \quad \text{with} \quad \sigma_{ij} = \mathcal{A}_{jilk}\, u_{k,l}}_{\text{Incremental Equilibrium Equations}}$$

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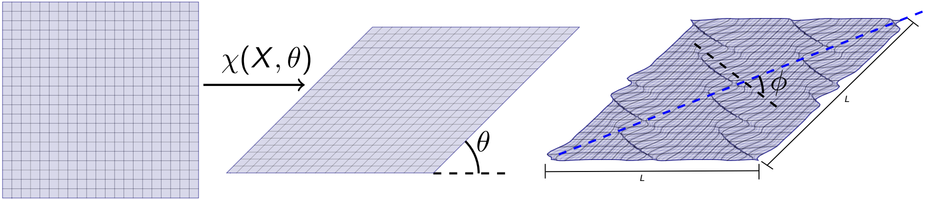

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$$\underbrace{\lim_{y \to \infty} u_{ij} \to 0}_{\text{Decay Condition}} \qquad \underbrace{\sigma_{21} = \sigma_{22} = \sigma_{23} = 0}_{\text{Zero Surface traction}}$$

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Mooney-Rivlin

$$W = \frac{\mu}{4}\left[(1+f)(I_1-3)+(1-f)(I_2-3)\right],$$

with

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  - Extreme Mooney-Rivlin $f = -1$ with the above σ_0
    $$\sigma_0^4 + \sigma_0^3 + \lambda_1^2\lambda_2^2\left(\lambda_1^4\lambda_2^4 - \lambda_2^2 - \lambda_1^2\right)\sigma_0(\sigma_0 + 1) + 4\lambda_1^6\lambda_2^6 = 0$$

Predictions

Mooney-Rivlin

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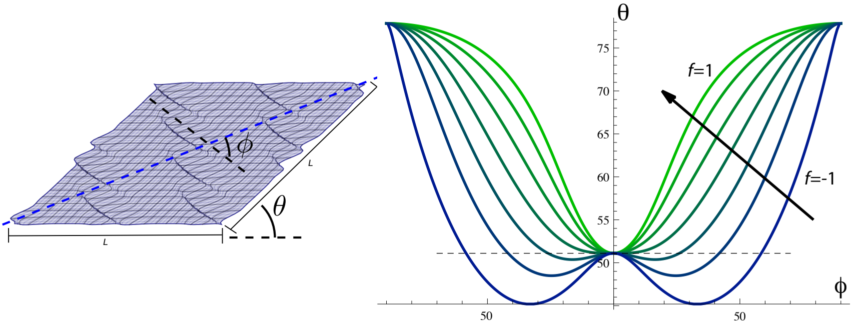

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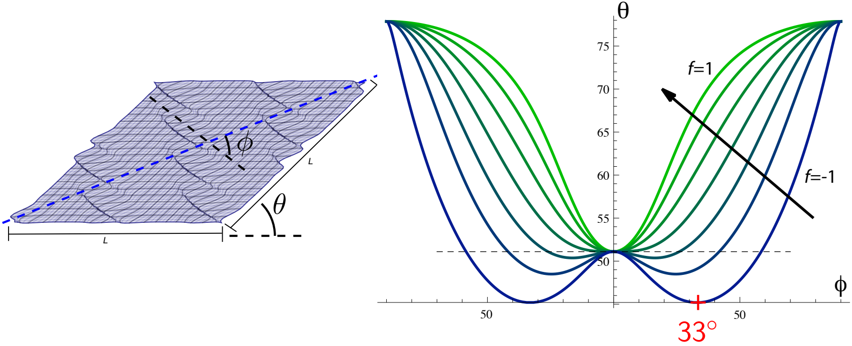

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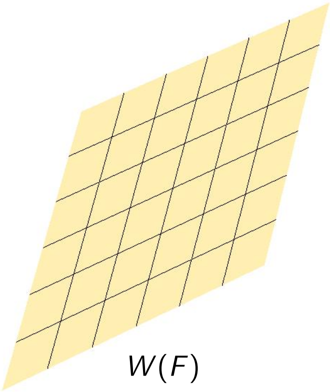

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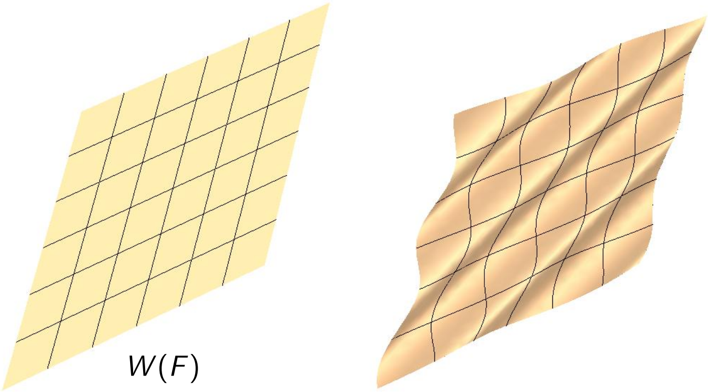

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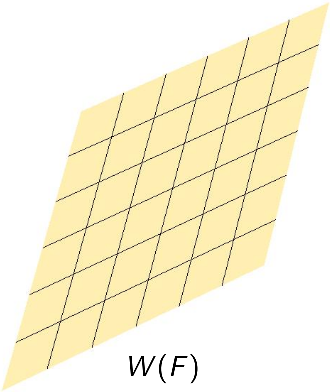

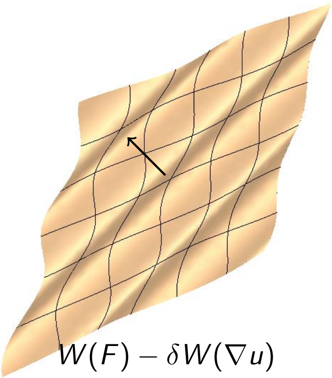

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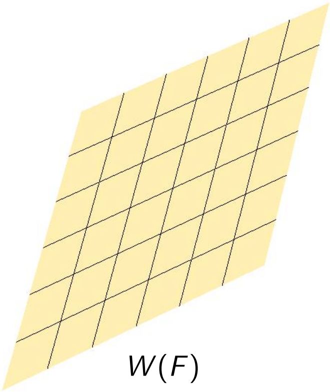

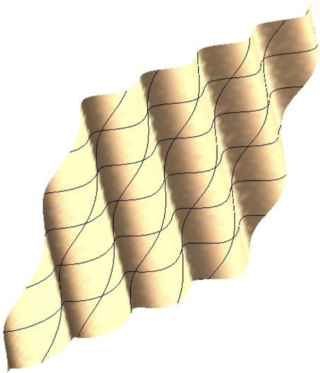

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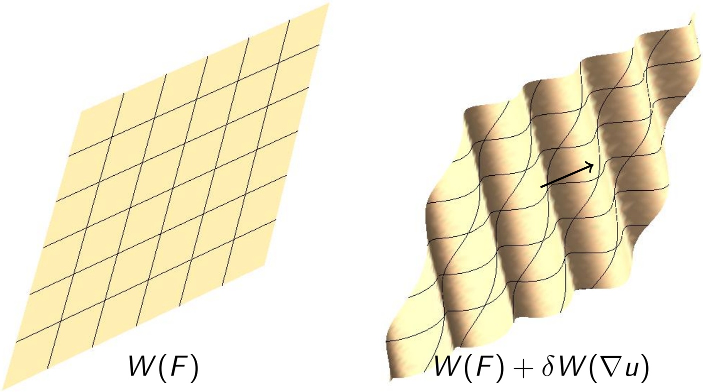
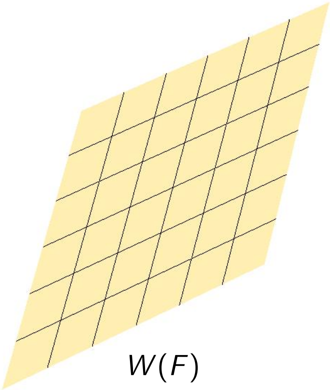


$W(F)$ $\qquad$ $W(F) + \delta W(\nabla u)$

Vague Energy Considerations

Similarly: zero traction $\implies$ zero surface energy increment.



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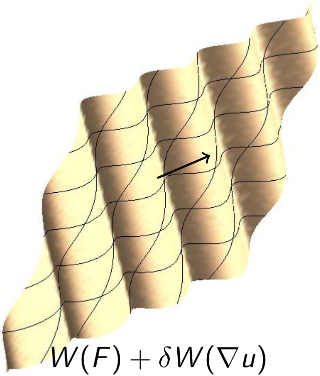

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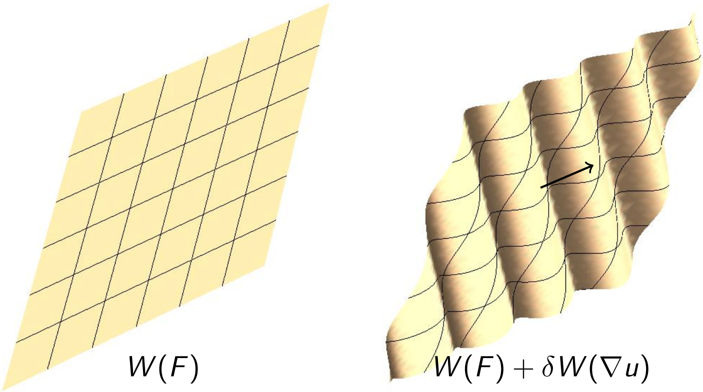
Similarily: zero traction $\implies$ zero surface energy increment.

$$0 = u_i^* \sigma_{i2} = \mathcal{A}_{jilk} u_{i,j}^* u_{l,k} = \delta W(\nabla u), \text{ on } y = 0.$$



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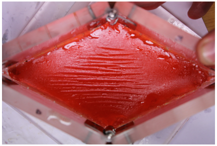

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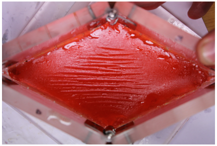

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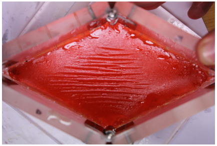

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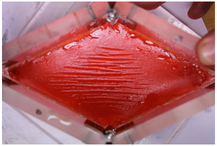

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Any questions?
Thanks for listening and hope you enjoyed the talk!

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