2S1 Problem Sheet 1

November 14th, 2003, Lecturer: Claas Röver

1. For the following functions determine their domains, decide where they are continuous and find their partial derivatives.

   $(a)\quad f(x,y) = \frac{x^2y}{x-y}$ $\qquad (b)\quad f(x,y) = x^2\sin y + y\cos x$

   $(c)\quad f(x,y) = \ln(xy)$ $\qquad (d)\quad f(x,y) = \frac{1}{3}e^{x^2+y^2}$

   $(e)\quad f(x,y) = 2|x| - 3|y|$ $\qquad (f)\quad f(x,y) = \sqrt{x^2+y^2}$

2. In your own words, describe what the parial derivatives of a function of two variables describes.

3. Determine the partial derivatives of the following functions

   $(a)\quad f(x,y) = \sin(xy - 3x)$ $\qquad (b)\quad f(x,y) = (x - 9y^2)e^{-3x+4y}.$

   $(c)\quad f(x,y) = (\ln x)\cos(2x^3)$ $\qquad (d)\quad f(x,y) = \frac{2e^{-x^2}}{x+y}$

4. State the condition for a function of two variables to be differentiable and give a verbal argument for the fact that a differentiable function must be continuous.

5. Let $x(t) = 3t$, $y(t) = -t^2$ and $f(x,y) = 4xy - 2x^2$. Find $\frac{\partial f}{\partial t}$.

6. Sketch the level curves of the following functions for four different values of your choice.

   $(a)\quad f(x,y) = x^2 + y^2$ $\qquad (b)\quad f(x,y) = 3x - y$

   $(c)\quad f(x,y) = x^3 + 2y$ $\qquad (d)\quad f(x,y) = xe^y$

7. Give a description of the plane in $\mathbb{R}^3$ which contains the three given points P, Q and R, and find the point where it intersects the y-axis.

   $(a)\quad P = (0,1,1),\quad Q = (1,2,0),\quad R = (1,1,2)$

   $(b)\quad P = (-1,0,2),\quad Q = (3,-3,0),\quad R = (2,-2,-1)$

   $(c)\quad P = (-2,1,0),\quad Q = (5,0,1),\quad R = (-1,4,2)$

8. For each plane in the previous problem find a unit vector orthogonal to the plane.

9. Find the extrem points, that is relative maxima and minima, of the following functions of two variables.

   $(a)\quad f(x,y) = 2x^2 + y^2 - xy + 7y$ $\qquad (b)\quad f(x,y) = -xye^{-\frac{1}{2}(x^2+y^2)}$

   $(c)\quad f(x,y) = y^2 - xy + 2x + y + 1$ $\qquad (d)\quad f(x,y) = x\sin y$

10. Describe in words what the gradient of a function of two variables is.

11. Determine the gradients of the functions in Problem 6.

12. (a) Find the directional derivative of $f(x,y) = \frac{5}{x^2+y^2}$ at the point P towards the origin when

$$(i) \quad P = (2,1) \qquad (ii) \quad P = (-1,2) \qquad (iii) \quad P = (\sqrt{5},0).$$

(b) Find the directional derivative of $f(x,y) = Ax^2 + Bxy + Cy^2$ at (a,b) towards (b,a), where A, B and C are constants.

13. Give a reason why you always got the same answer in part (a) of the previous problem.

14. What is the maximal volume of a rectangular box which fits into the sphere of radius r; the sphere is given by $x^2 + y^2 + z^2 = r^2$.