3E1 Problem Sheet 1

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1. Determine the Fourier coefficients a_0, a_n and b_n, $n \geq 1$, for the following functions $f(x)$.

   (a) $f(x) = \frac{1}{3} \sin x$

   (b) $f(x) = \cos 4x$

2. Sketch the graph of the 2π-periodic function $f(x)$ which is determined by
$$f(x) = \begin{cases} 1 + \frac{x}{\pi}, & -\pi < x \leq 0 \\ 1 - \frac{x}{\pi}, & 0 \leq x < \pi \end{cases}$$
   and find its Fourier coefficients. (Show your working!) Thus express $f(x)$ in the form $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$.
   Hint: $\int x \cos nx \, dx$ can be computed using integration by parts.

3. Suppose that $f(x)$ and $g(x)$ are 2π-periodic functions with Fourier coefficients $a_0^{(f)}$, $a_n^{(f)}$ and $b_n^{(f)}$ and $a_0^{(g)}$, $a_n^{(g)}$ and $b_n^{(g)}$ respectively, where $n \geq 0$

   (a) Prove that $h(x) = \mu f(x) + \nu g(x)$ is a 2π-periodic function for all real constants μ and ν.

   (b) Can you give a formula for the Fourier coefficients of $h(x)$ in terms of those for $f(x)$ and $g(x)$?

   (c) Prove that $k(x) = f(x)g(x)$ is a 2π-periodic function.

   (d) Can you give a formula for the Fourier coefficients of $k(x)$ in terms of those for $f(x)$ and $g(x)$?

4. (Optional home work) Use a graphing utility, eg. powerful calculator or Mathematica, to plot graphs for the first five partial sums of the Fourier series of the triangle function obtained in question 2. The partial sums are obtained by taking only finitely many initial terms of the infinite sum $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$.
   The idea is to see how $f(x)$ gets approximated more accurately with each additional summand.