

# 3E1 Problem Sheet 10

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1. (a) Let  $n$  be an integer. Show that if  $z$  is a complex number with  $z^n = 1$ , then  $\bar{z}^n = 1$  as well, where  $\bar{z}$  denotes the complex conjugate of  $z$ .

Hint: Use the polar coordinate representation of  $z$ .

- (b) Find *all* complex numbers  $z$  satisfying  $z^3 = 1$  and draw them in the complex plane. Give an argument why, if you sum them all up you get zero.

Hint: The cosine of 60 degrees is  $\frac{1}{2}$ .

2. Let  $f(z)$  be an analytic function. Prove that, if  $\text{Im}(f(z)) = c = \text{const.}$ , then  $f(z) = C = \text{const.}$
3. Decide at which points the function  $f(z) = |z|$  is differentiable. (Recall that the absolute value  $|z|$  is defined by  $\sqrt{x^2 + y^2}$  when  $z = x + iy$ .)