3E1 Problem Sheet 10

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1. (a) Let n be an integer. Show that if z is a complex number with $z^n = 1$, then $\overline{z}^n = 1$ as well, where $\overline{z}$ denotes the complex conjugate of z.
   Hint: Use the polar coordinate representation of z.

   (b) Find *all* complex numbers z satisfying $z^3 = 1$ and draw them in the complex plane. Give an argument why, if you sum them all up you get zero.
   Hint: The cosine of 60 degrees is $\frac{1}{2}$.

2. Let $f(z)$ be an analytic function. Prove that, if $\mathrm{Im}(f(z)) = c = const.$, then $f(z) = C = const.$

3. Decide at which points the function $f(z) = |z|$ is differentiable. (Recall that the absolute value $|z|$ is defined by $\sqrt{x^2 + y^2}$ when $z = x + iy$.)