3E1 Problem Sheet 12

February 16-22, 2004

Lecturer: Claas Röver

1. Find the images of the following curves under the mapping $f(z) = z^3$.

   (a) Rays emanating from the origin.

   (b) Circles centred at the origin.

   Your answer should be like "the image of the ray emanating from the origin at an angle θ with the x-axis is ... because ..." and similarly for circles.

2. State what it means that a complex function is conformal in a domain D in the complex plane.
   Then decide where the mapping $f(z) = z^3$ is conformal and justify your answer.

3. Define two parametric curves as follows.

$$\begin{array}{lll} C_1: & z_1(t) = t + i, & 0 \leq t \leq 2 \\ C_2: & z_2(s) = s + is^2, & 0 \leq s \leq 2 \end{array}$$

   (a) Find the point w where the two curves intersect if it exists and draw the two curves indicating the direction of increasing parameter by an arrow.

   (b) Find tangent vectors to the two curves at the point w and draw them into the sketch from part (a).

   (c) Find the image $f(w)$ of the intersection point w under the mapping $f(z) = z^3$ and draw tangent vectors to the images of the two curves at this point. Justify your answer.