3E1 Problem Sheet 13

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1. State the definition of a simply connected subset of the complex plane. Sketch the following subsets of $\mathbb{C}$ and decide whether they are simply connected; justify your answer

   (a) $\{z \in \mathbb{C} \,|\, 1 < |z| < 2\}$

   (b) $\{z \in \mathbb{C} \,|\, 1 < |z| < 2,\ 0 \le \arg(z) \le \pi\}$

   (c) $\{z \in \mathbb{C} \,|\, z \neq 0, |z| \le 3\}$

2. Evaluate the following complex integrals, where C is the unit circle traversed counterclockwise from 1.

$$(a) \quad \int_C 2z^2 + 3z - i\,dz \qquad\qquad (b) \quad \int_C \mathrm{Re}(z)\,dz$$

   Hint: $\cos^2 \alpha = \frac{1}{2}(1 + \cos(2\alpha))$.

3. Let $f(z) = \frac{1}{z^2}$. Decide where $f(z)$ is analytic and compute the integral

$$\int_C f(z)\,dz,$$

   where

   (a) C is the upper semi-circle of radius r centred at the origin and traversed counterclockwise,

   (b) C is the lower semi-circle of radius r centred at the origin and traversed clockwise,

   (c) C is the circle of radius r about the origin traversed counterclockwise.