3E1 Problem Sheet 14

March 1-7, 2004

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1. Use Cauchy's integral formula to evaluate the integral

$$\int_C \frac{z^2}{z^2+1}\,dz,$$

where C is the counterclockwise oriented circle of radius 1 centred at i.

2. Use Cauchy's integral formula to evaluate the integral

$$\int_C \frac{e^z}{z^3}\,dz,$$

where C is the counterclockwise oriented circle of radius 1 centred at the origin.

3. State the definition of a harmonic real function of two real variables. Then verify that

$$u(x,y) = x - \frac{y}{x^2+y^2}$$

is harmonic in $\mathbb{R}^2 \setminus \{0\}$ and find a complex function whose real part is equal to $u(x,y)$ and which is analytic in $\mathbb{C} \setminus \{0\}$.