# 3E1 Problem Sheet 15

March 29 – April 4, 2004

Lecturer: Claas Röver

1. Find the centre and the radius of convergence of the following power series

   (a)
   $$1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}(z-2)^n$$

   (b)
   $$\sum_{n=0}^{\infty} n^3 z^n$$

   (c)
   $$\sum_{n=0}^{\infty} \left(\frac{4-2i}{1+5i}\right)^n (z-1-i)^n$$

2. (a) Determine the Taylor series of $f(z) = \cos z$ with centre $\pi$.

   (b) Recall that the Maclaurin series of
   $$f(z) = \frac{1}{1-z} \quad \text{is} \quad \sum_{n=0}^{\infty} z^n.$$
   Use this to determine the Maclaurin series of
   $$(i) \quad g(z) = \frac{1}{z+1} \quad \text{and} \quad (ii) \quad h(z) = \frac{1}{(z-1)^2}.$$

   (c) Why is there no Maclaurin series for $f(z) = \frac{1}{z}$.

3. Let $f(z) = \frac{-z}{z+1}$. Show that for every complex number $z$ with negative imaginary part, the imaginary part of $f(z)$ is positive.