3E1 Problem Sheet 17

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1. Consider the network N in Fig. 1 where an edge label c, f means that the corresponding edge has capacity c and carries the flow f.
   (a) What is the total flow in N?
   (b) Find a flow augmenting path in N; give its vertices in order. By how much can you increase the total flow in N at most using that flow augmenting path?
   (c) What is the capacity of the cut set consisting of the edges $(2, 3)$, $(3, 6)$, $(6, 4)$, $(5, 8)$ and $(8, 9)$?

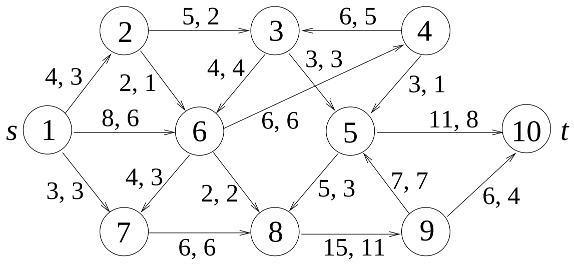

FIGURE 1

2. Solve the following linear optimisation problem geometrically.
   Maximise $f(x_1, x_2) = 20x_1 + 30x_2$ subject to the constraints

(1) $4x_1 + 3x_2 \geq 12$	(2) $x_1 - x_2 \geq -3$
(3) $x_2 \leq 6$	(4) $2x_1 - 3x_2 \leq 0$

3. A company produces three types of glass, flint glass (G_1), German crystal-glass (G_2) and crown-glass (G_3). The amount of raw material (in kg) needed for $200kg$ of glass and the price per kg (in Euro), depending on the type, are given in the table below.

	sand	pearlash	arsenic	red-lead	nitre	price
G_1	120	35	6	40	13	35
G_2	120	46	6	0	7	60
G_3	120	60	1	0	30	70

   Per day they have $2000kg$ of sand, $450kg$ of pearlash, $50kg$ of arsenic, $150kg$ of red-lead and $180kg$ of nitre available.
   Formulate a constrained optimisation problem in order to maximise the daily revenue, assuming all the glass can be sold, and transform it into normal form, by possibly introducing slack variables.