3E1 Problem Sheet 2

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1. Let $f(x) = Ae^{\mu x} + Be^{-\mu x}$, $\mu \neq 0$.

   (a) Show that $f(x)$ is a solution of the ordinary differential equation
   $$\frac{d^2u}{dx^2} - \mu^2 u = 0.$$

   (b) Prove that, if $f(0) = f(L) = 0$ for some positive real number L, then $A = B = 0$.

2. Let $f_n(x) = \sin nx$, where n is a positive integer.

   (a) Sketch, in one figure, the graphs of the functions
   $$f_1(x), \quad -\frac{1}{2}f_2(x) \quad \text{and} \quad \frac{1}{4}f_4(x)$$
   for values of x with $0 \leq x \leq 2\pi$.

   (b) What is the elementary period and the maximum height of $Kf_N(x)$ where N is a positive integer and K any real number?

   (c) With the help of part (a), sketch the graph of the function
   $$f(x) = \sin x - \frac{1}{2}\sin 2x + \frac{1}{4}\sin 4x.$$

3. Recall from the very first lecture that the 2π-periodic function determined by
   $$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$$
   where $k > 0$ is constant, is given by the Fourier series
   $$\frac{k}{2} + \frac{2k}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin(2n-1)x.$$
   Use this to obtain the Fourier series of the 2π-periodic function $g(x)$which is determined by
   $$g(x) = \begin{cases} -1, & -\pi < x < -\frac{\pi}{2} \\ 1, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ -1, & \frac{\pi}{2} < x < \pi \end{cases}$$
   Justify your answer.